Level-0 Models for Predicting Human Behaviour in Games

Behavioural Game Theory

- Sometimes game theory recommends actions that seem counter-intuitive
- Example: “Travellers Dilemma“
- **Do people actually follow them?**

Level-k

"Player types are drawn from a hierarchy of smartness analogous to the levels of iterated rationalizability"

-Stahl, D. O. (1993). Evolution of Smart$_n$ Players

Level-k


...
LEVEL 2
Thinks everyone is Level 1
LEVEL 1
Thinks everyone is Level 0
LEVEL 0
The Non-Strategic Player

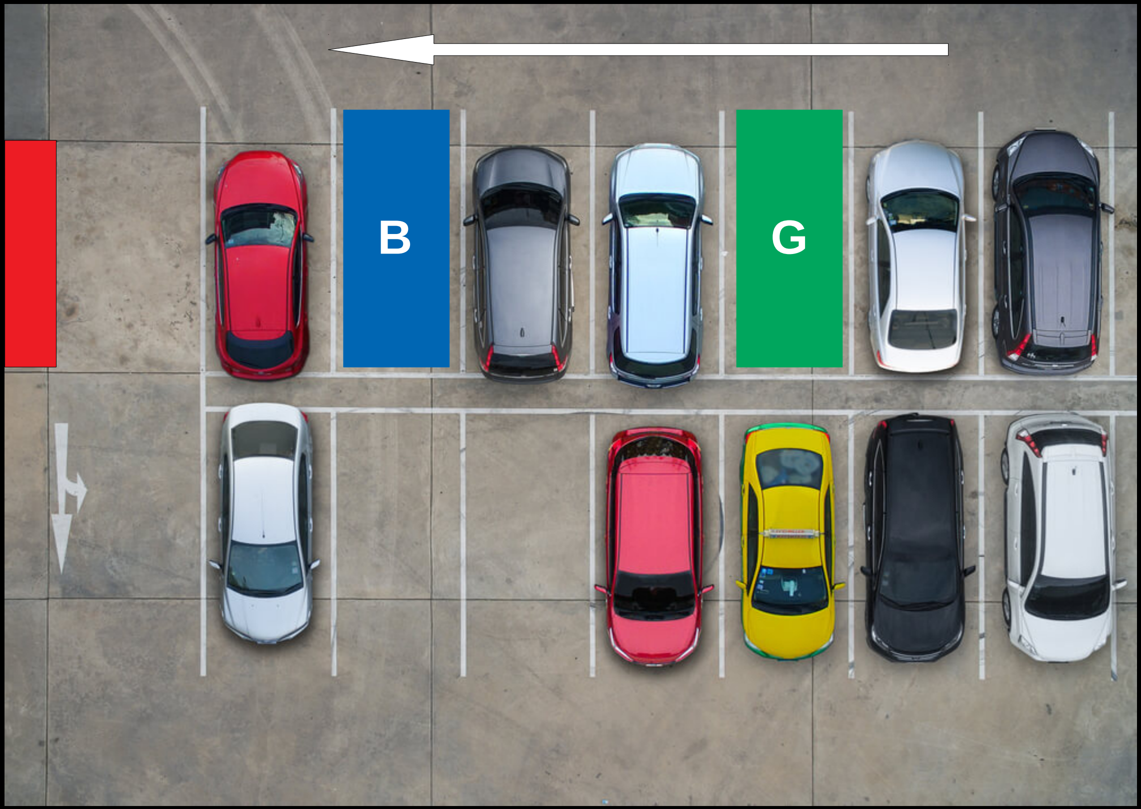
Cognitive Hierarchy

- A Player does not necessarily fall under one of these archetypes
- Assumptions can be made about mixed populations
- E.g. 50% Level-0, 50% Level-1

- **Is this a model for human behaviour?**

Quantal Cognitive Hierarchy

- Cognitive Hierarchy doesn‘t account for human mistakes
- Humans don‘t always go for the best response



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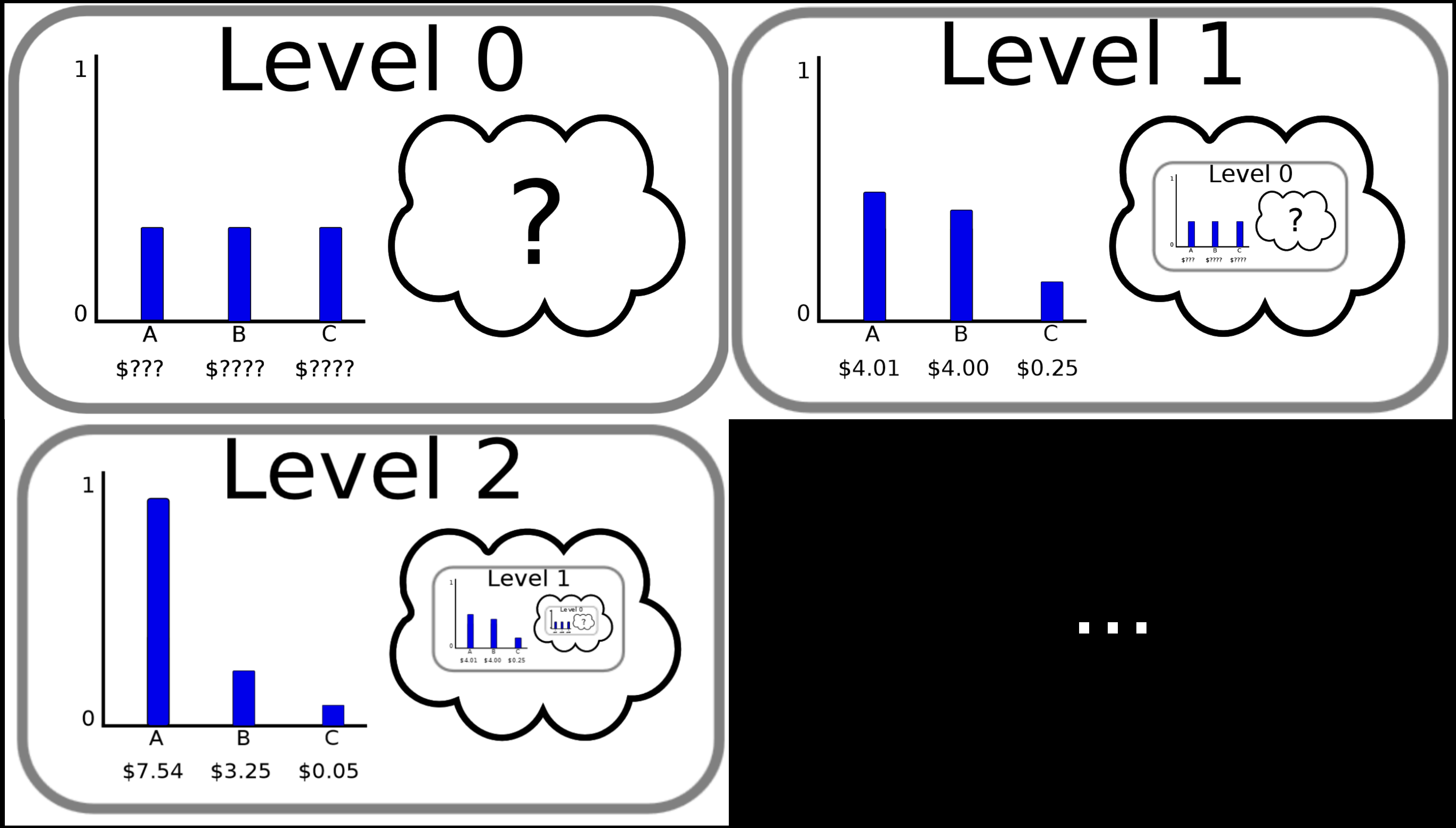
Quantal Best Response

QBR$_i$(s$_{-i}$;G, λ) always returns a single mixed strategy **s$_i$**

$$s_i(a_i) = \frac{\exp[\boxed{\lambda} * \boxed{u_i(a_i, s_{-i})}]}{\sum_{a'_i \in A_i} \exp[\boxed{\lambda} * \boxed{u_i(a'_i, s_{-i})}]}$$

- **u$_i$ (a$_i$, s$_{-i}$)** = expected utility of **agent i** when playing action **a$_i$** against mixed strategy profile **s$_{-i}$**
- **λ** = Precision → Agents Sensitivity to utility differences

Iterative reasoning



Quantal Cognitive Hierarchy

- **Poisson-QCH model:**

$$\pi_{i,0:m} = \sum_{l=0}^{m} \frac{Poisson(l;\tau)}{\sum_{l'=0}^{m} Poisson(l';\tau)} \pi_{i,l}$$

- The truncated distribution over actions predicted for an agent of level $0 \leq l \leq m$

Quantal Cognitive Hierarchy

- **Predicted action distribution:**

$$\boxed{\pi_{i,0}(a_i) = \left|A_i^{-1}\right|}$$

$$\pi_{i,m}(a_i) = QBR_i(\pi_{-i,0:m-1}; \lambda)$$

- Two parameters: **λ** (precision) and **τ** (mean of Poisson distribution)

Experiment

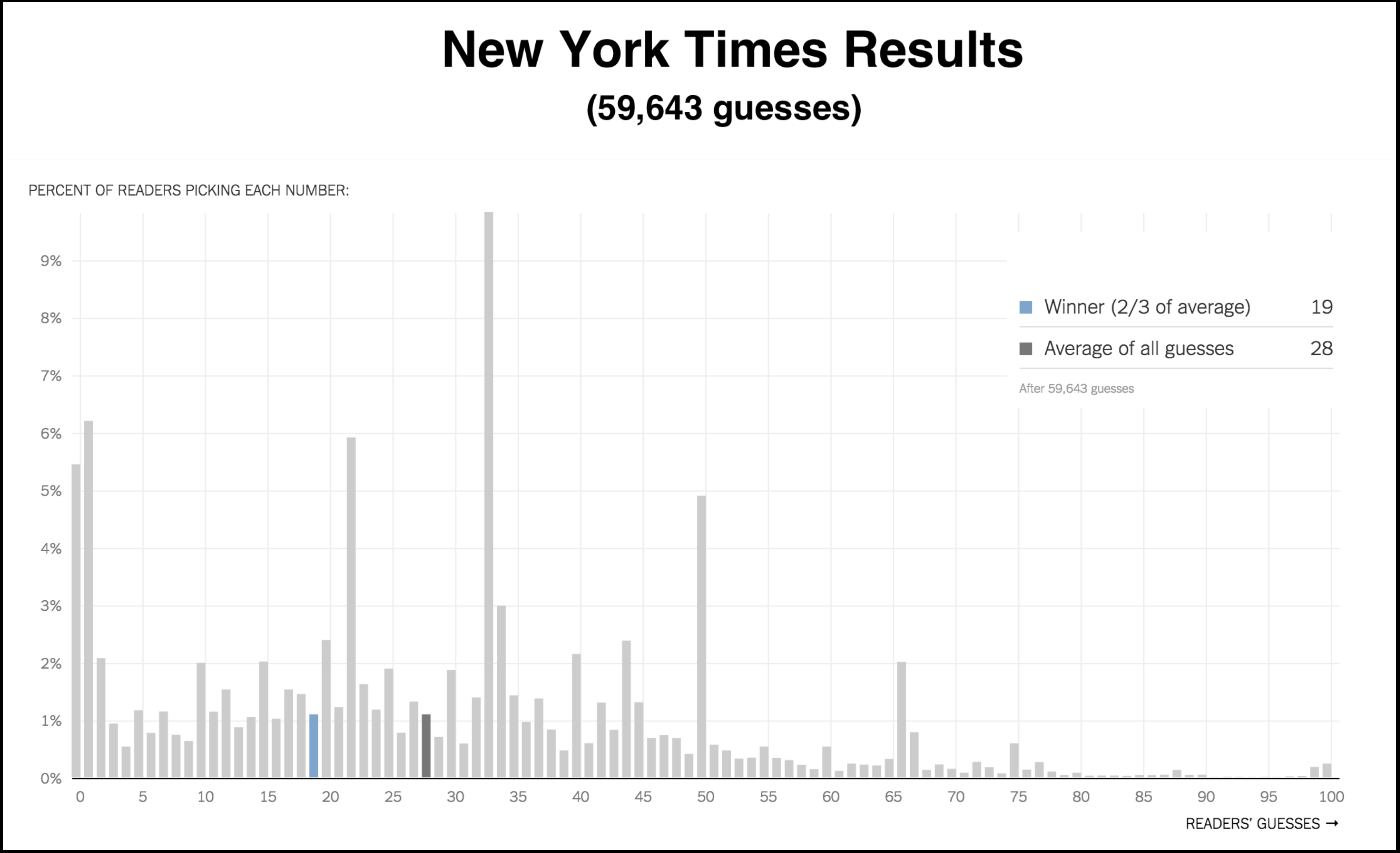
Pick a number **from 0 to 100**,
with that number representing your best guess of **two-thirds of the average** of all chosen numbers.

For example: Is the average of all numbers **63**, you would win by picking **42**. (No decimals or fractions)

If the average is **40**, you'd win by picking **27**.

Reason about the other people!

Quantal Cognitive Hierarchy



[4] nytimes.com/interactive/2015/08/13/upshot/are-you-smarter-than-other-new-york-times-readers.html

What’s the problem with current models?

Level 0 **Level > 0**

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Level-0 Models

I. What is non-strategic behaviour?

II. What are Level-0 Features?

III. How do we select a Model?

Non-strategic behaviour

- It doesn‘t have to be uniform!
- May take account of payoffs
- Not responding to explicit beliefs about other agents behaviour

  → Level-1 and higher = strategic
- can be computed via a finite combination of elementary agent models

Non-strategic behaviour

Elementary Agent Model:

An agent model for agent i is a function $f_i(G)$ that maps from a normal-form game G to a vector of reals with dimension $|(A_i)|$.

An agent model is $\boxed{\text{elementary}}$ if it can be computed as $f_i(G)=h_i(\Phi(G))$, where:

i) $\Phi(G)_a=\phi(u(a))$ for every action profile a,

ii) $\phi(u(a))=w^T u(a)$. $\phi(u(a))$ is computed by taking a linear combination of the players utilities at pure action profile a with the weights defined by a vector w in $\mathbb{R}^n$

Level-0 Features

- The models are driven by certain rules ("features")
- One or more actions are recommended to greater or lesser degree
- Can be computed directly form the normal form

Level-0 Features

1. **Maxmin payoff** – The best worst case
2. **Maxmax payoff** – The best best case
3. **Minimax regret** – The minimal maximal regret
4. **Maxmax fairness** – The “fairest” action
5. **Max symmetric** – The best response to oneself
6. **Maxmax welfare** – The best sum of utilities

We define a binary- and a real-valued version of each feature!

Level-0 Feature combination

F = set of features.

$w_f \in [0,1]\ with \sum_{f \in F} w_f \leqslant 1$

$w_0 = 1 - \sum_{f \in F} w_f$

Weighted Linear level-0 specification

$$\pi_{i,0}^{linear,F}(a_i) = \frac{w_0 + \sum_{f \in F} w_f f(a_i)}{\sum_{a'_i \in A_i} [w_0 + \sum_{f \in F} w_f f(a'_i)]}$$

Logit level-0 specification

$$\pi_{i,0}^{logit,F}(a_i) = \frac{\exp(w_0 + \sum_{f \in F} w_f f(a_i))}{\sum_{a'_i \in A_i} \exp(w_0 + \sum_{f \in F} w_f f(a'_i))}$$

Level-0 Features informativeness

- Are all these features always relevant?
- Do we always get a 'good' recommendation?

Player 2

Player 1

	A		B		C	
X	100	20	10	67	30	40
Y	40	35	49	50	90	70
Z	40	21	42	22	41	23

Level-0 Features informativeness

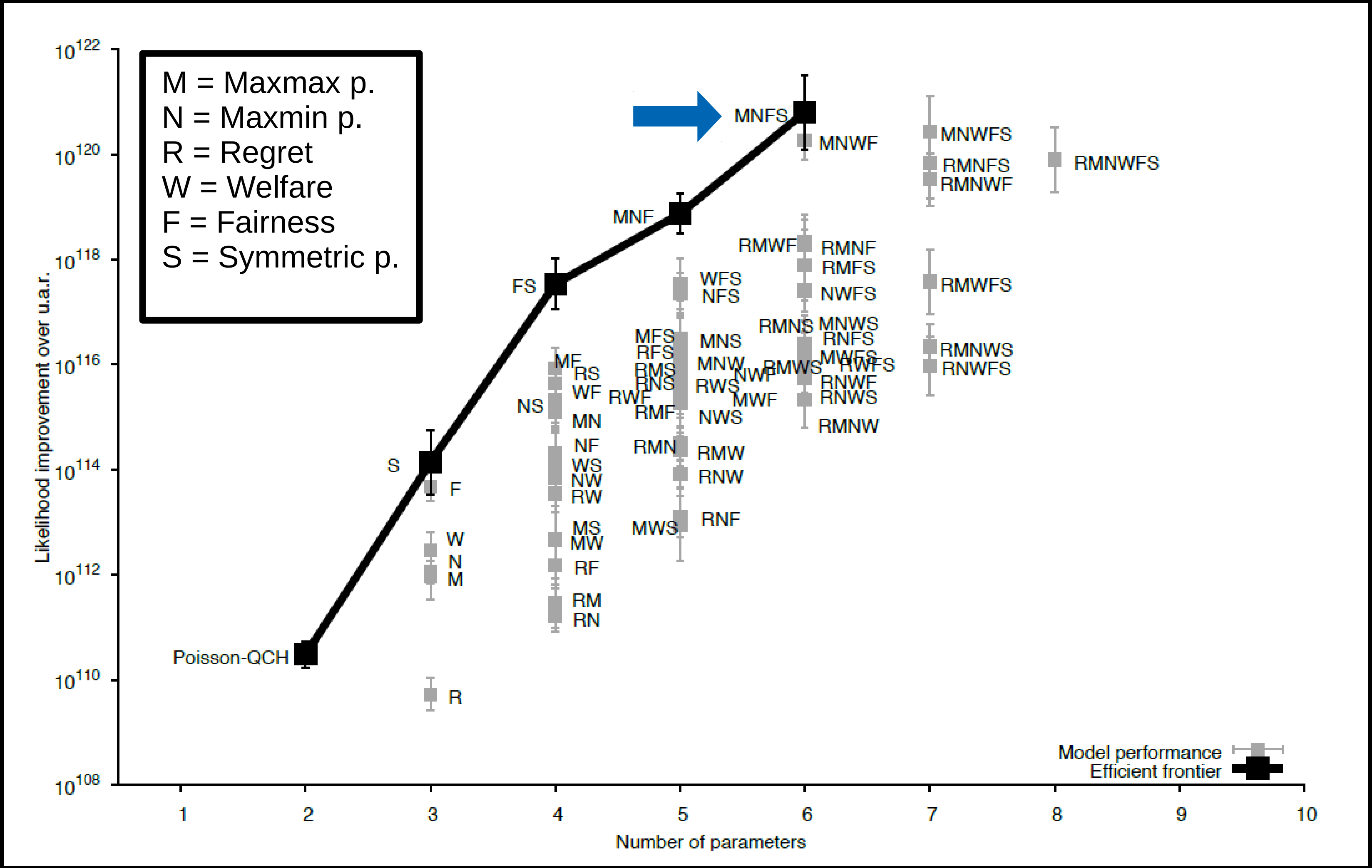
Player 2

Player 1

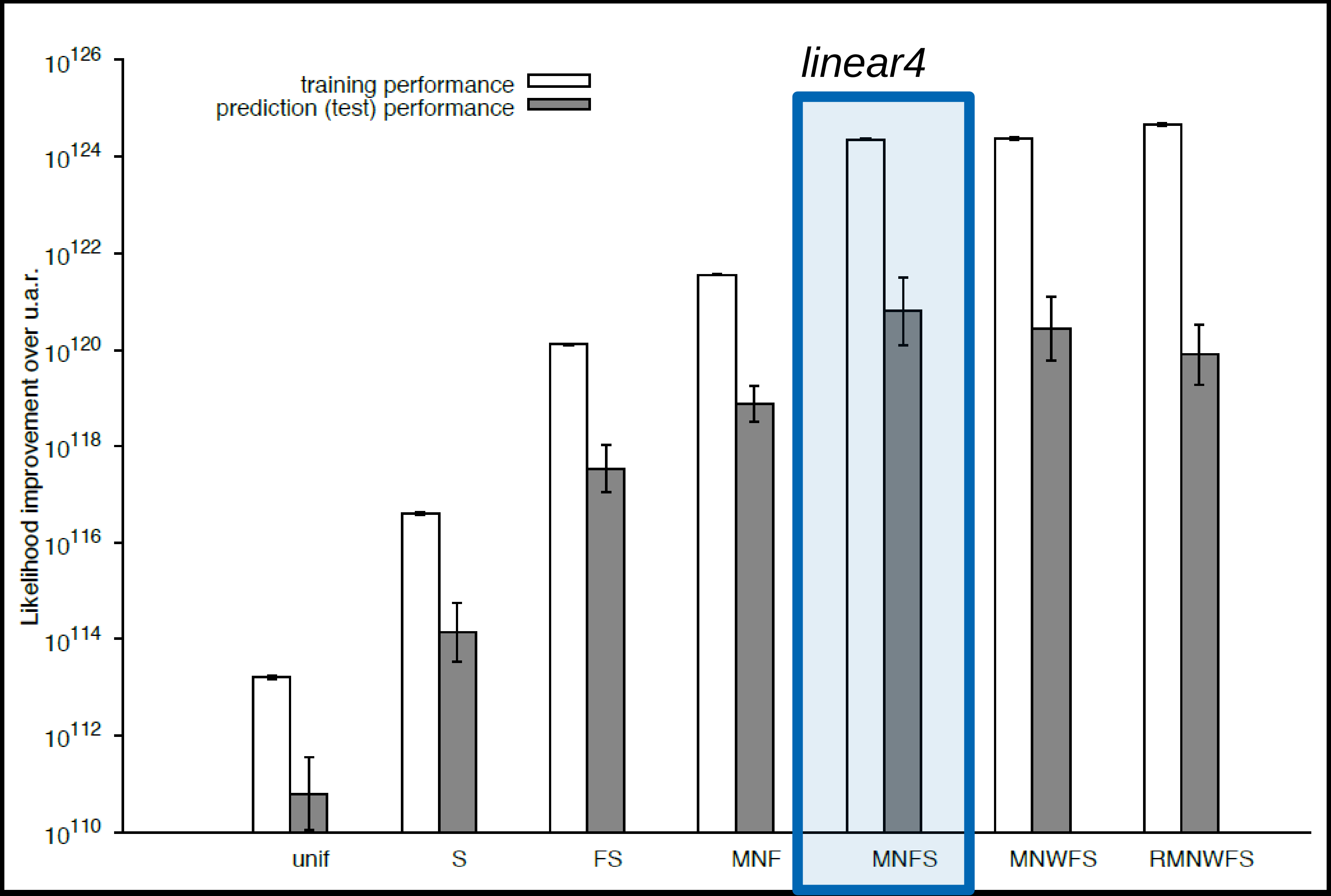
	A	B	C
X	100 20	10 67	30 40
Y	40 35	49 50	90 70
Z	40 21	42 22	41 23

1) Maxmin payoff
2) Minimax regret

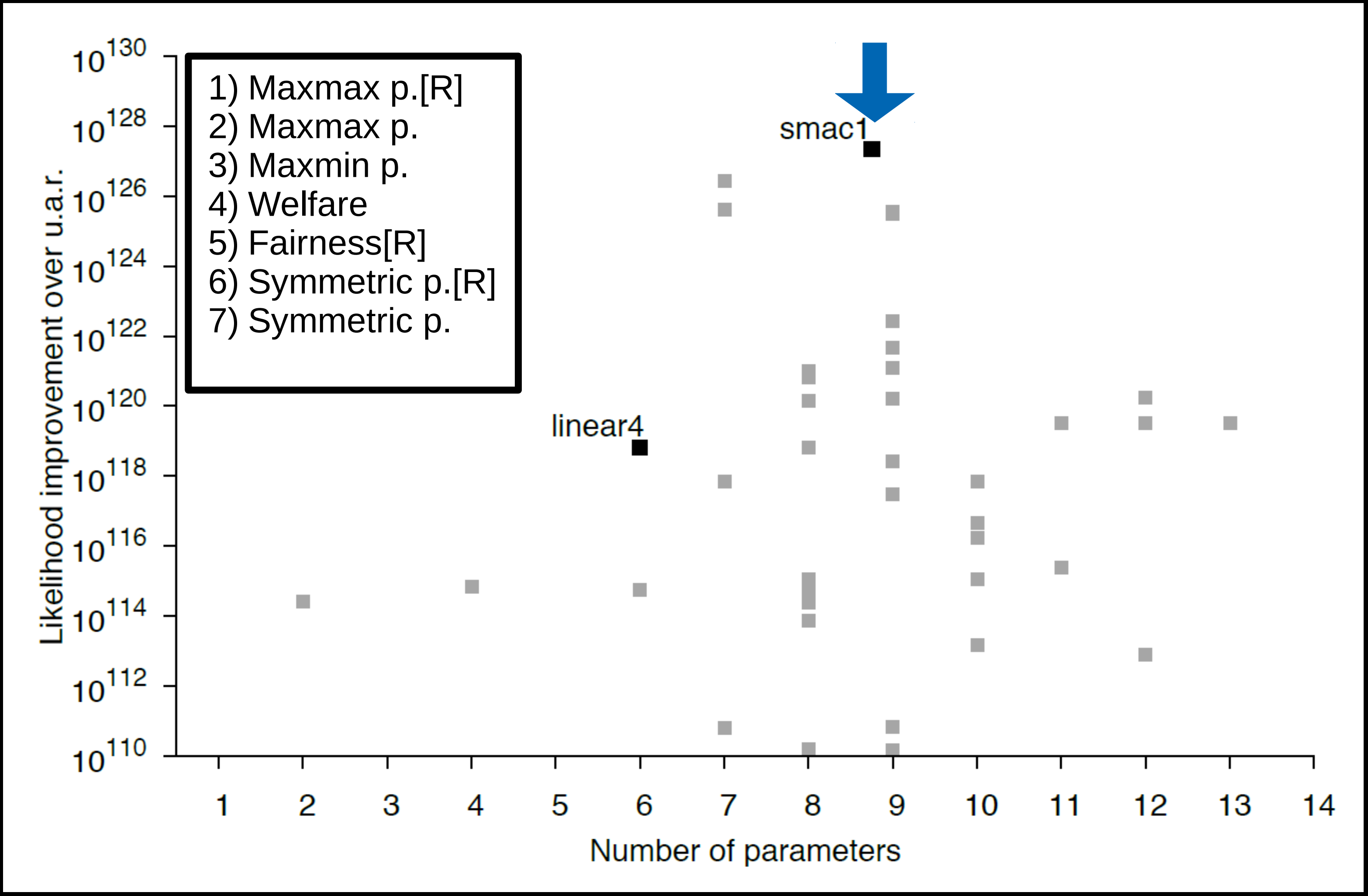
Model selection



Model selection

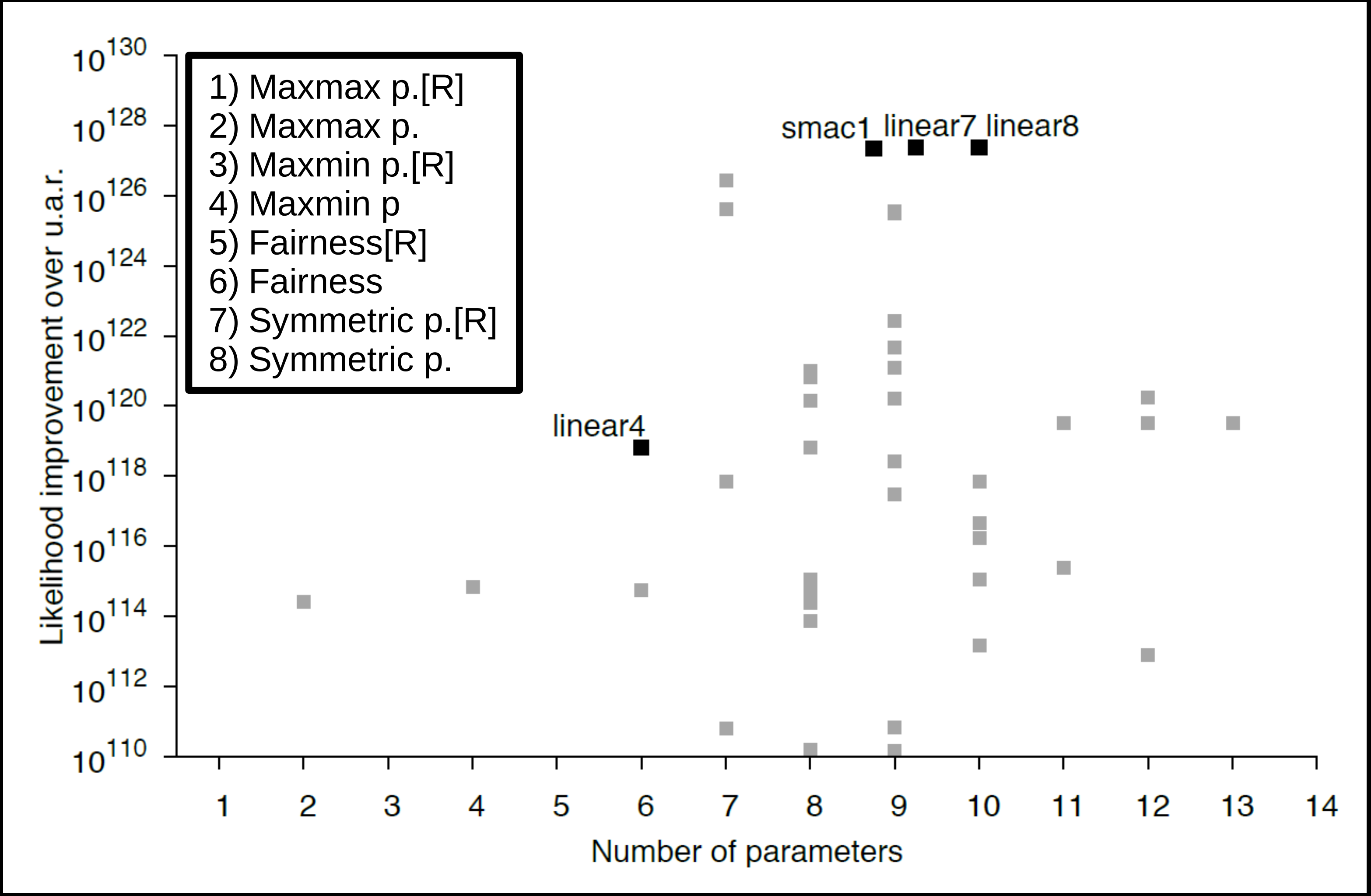


Model selection (Bayesian optimization)



First random training/test split

Model selection (Bayesian optimization)



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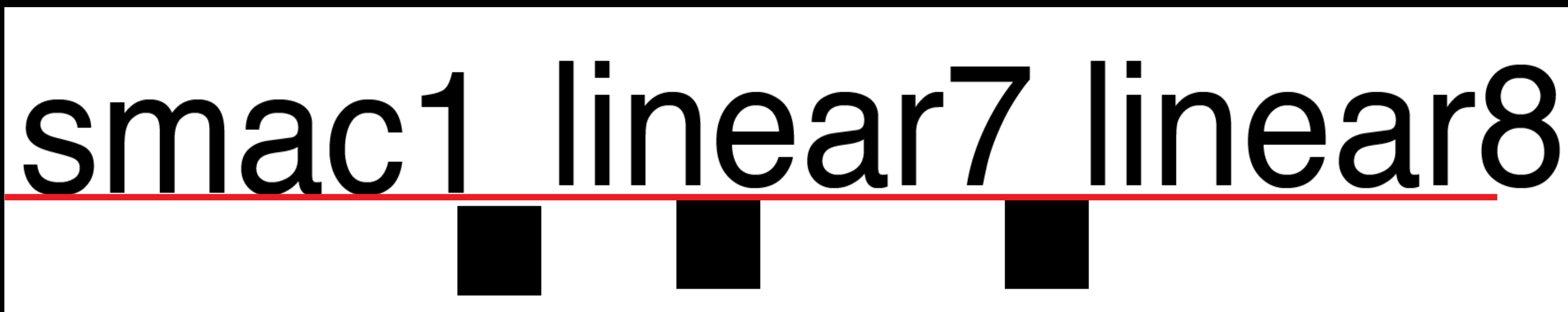

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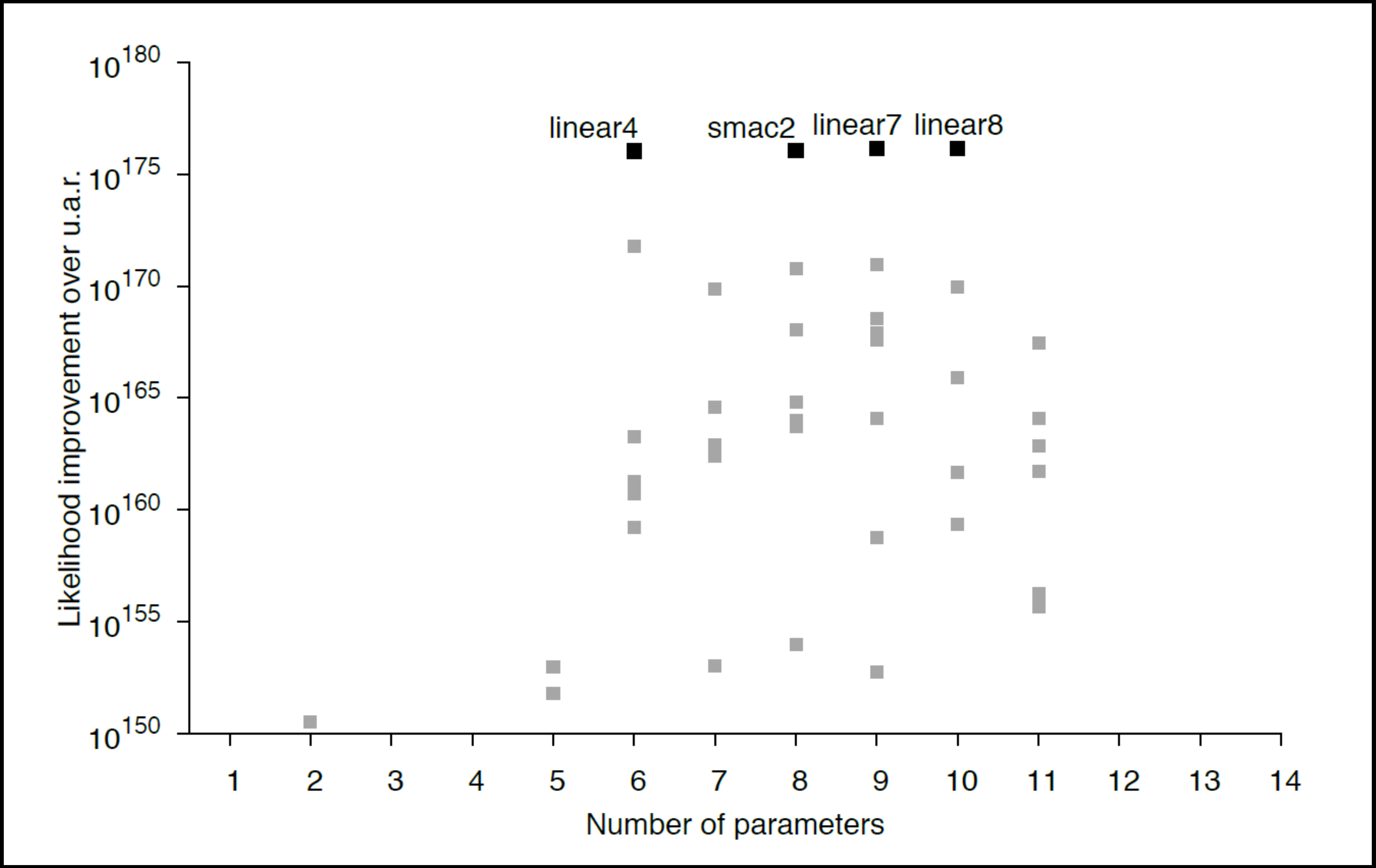

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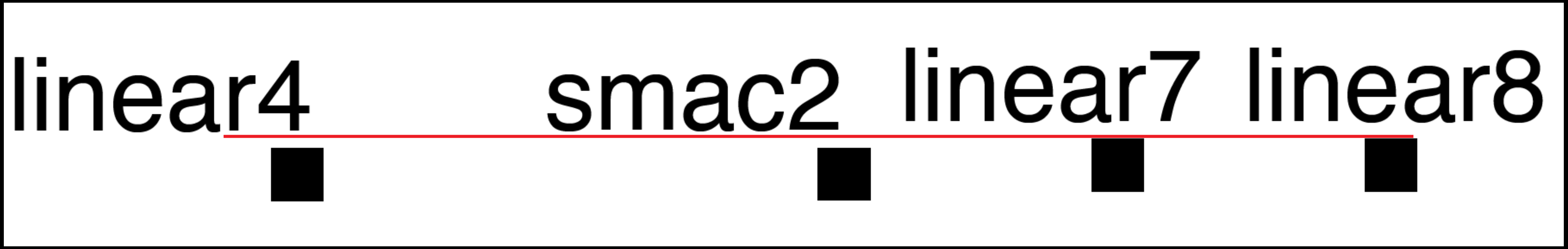
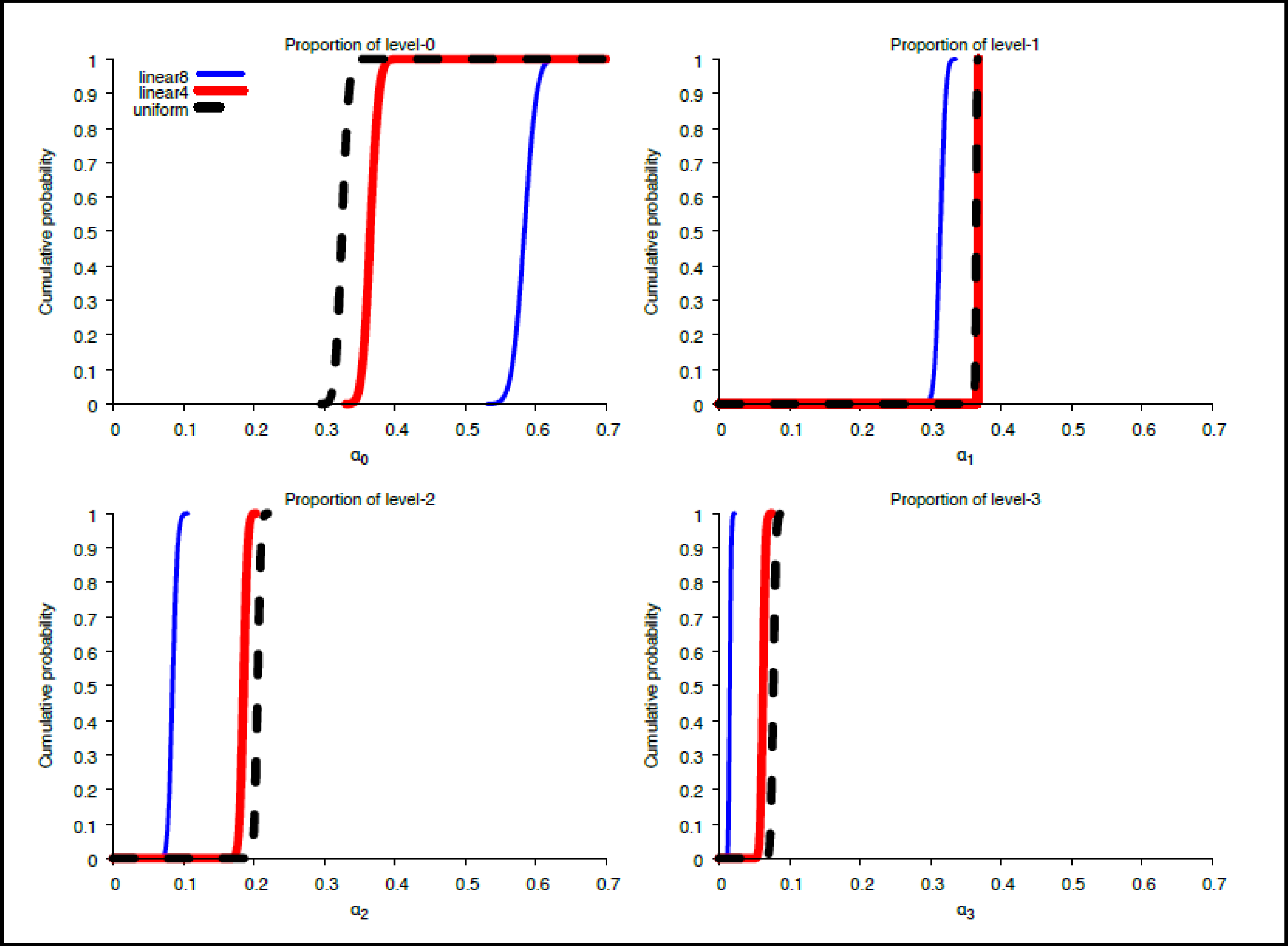


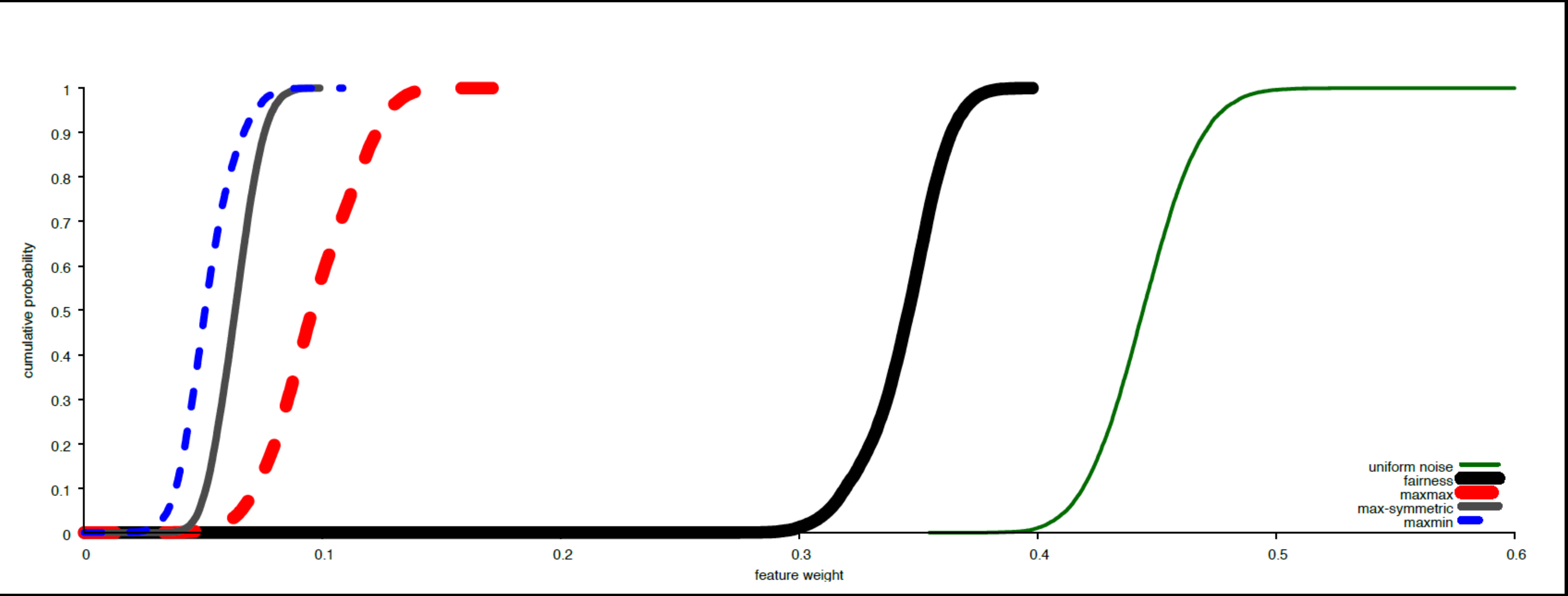
Parameter Analysis



Marginal cumulative posterior distributions of levels of reasoning

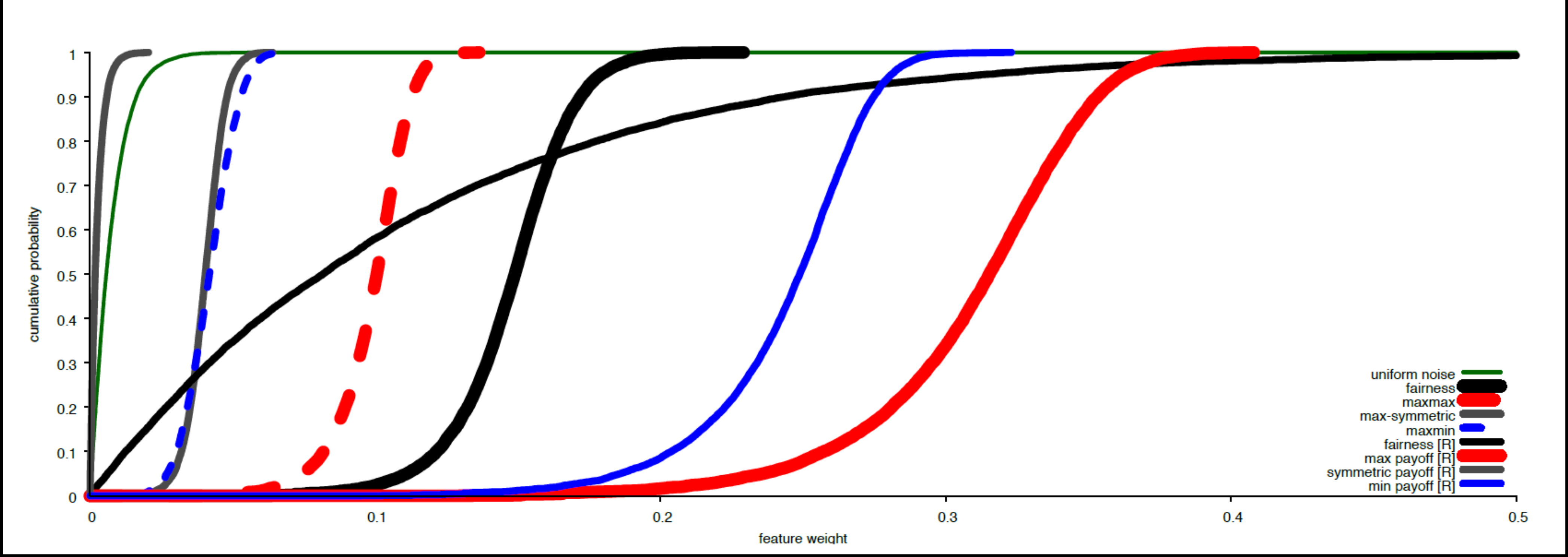
Parameter Analysis

Features in Linear 4

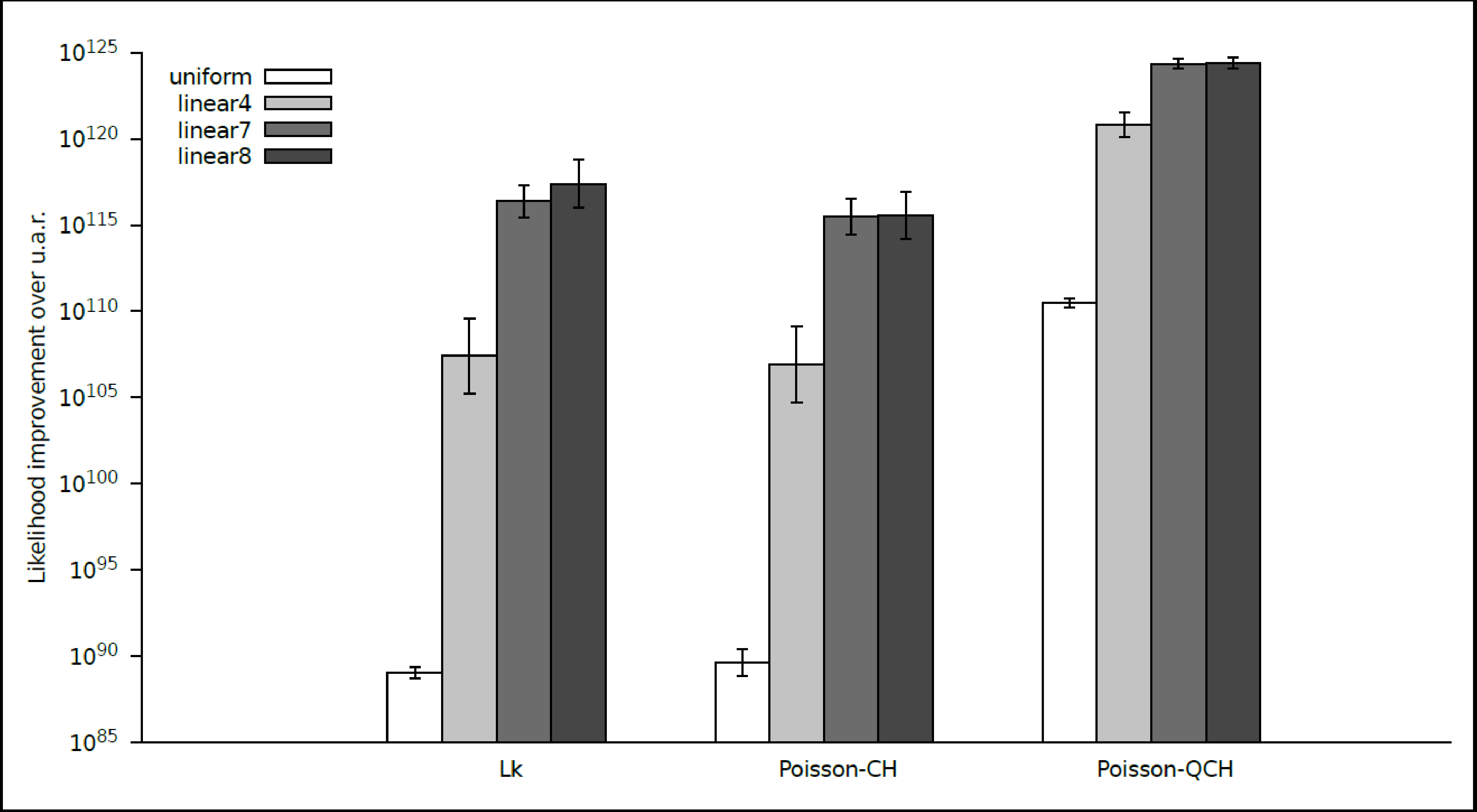


Parameter Analysis

Features in Linear 8



Conclusion



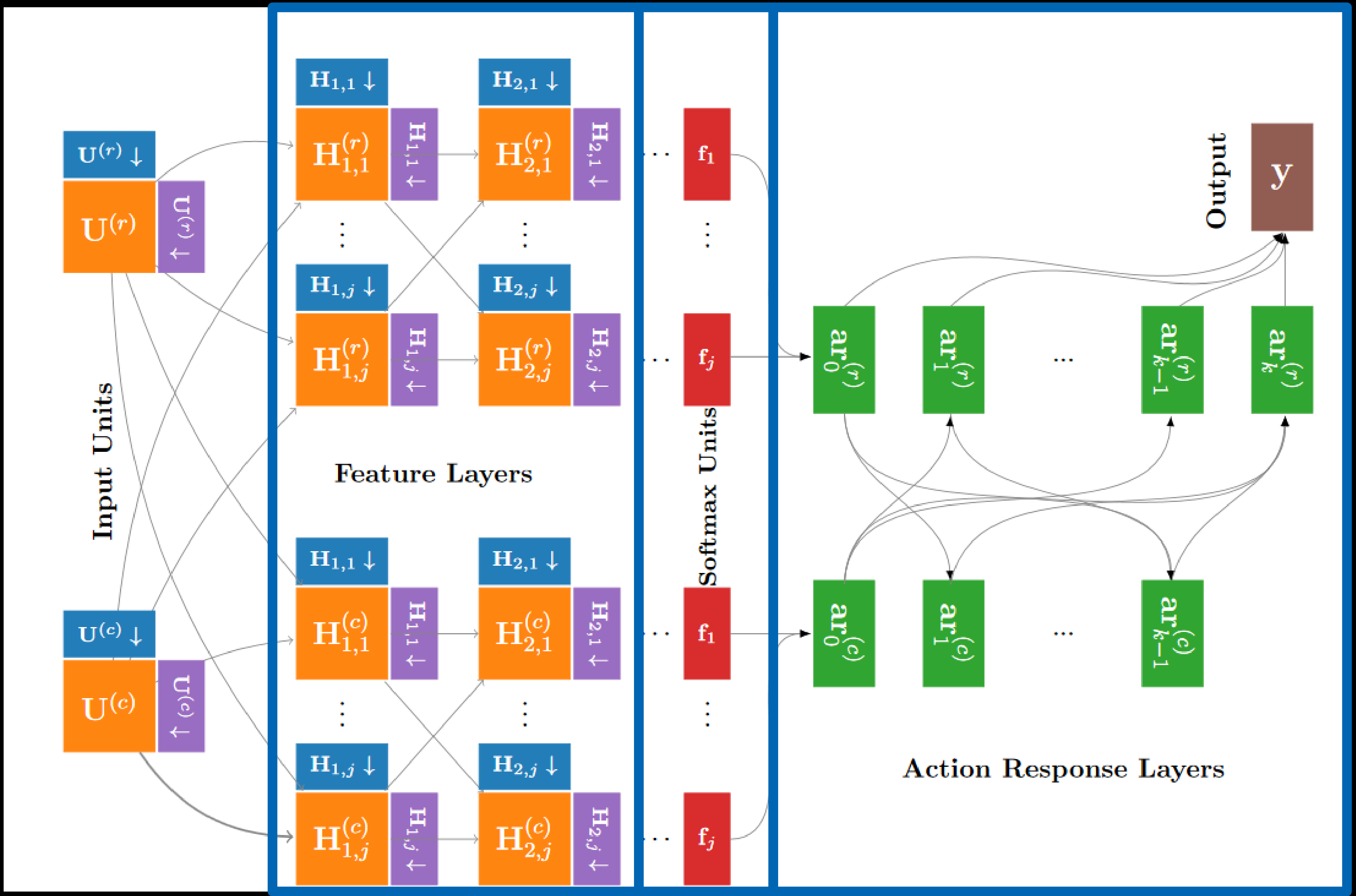
10125
10120
10115
10110
10105
10100
1095
1090
1085
uniform
linear4
linear7
linear8
Likelihood improvement over u.a.r.
Lk
Poisson-CH
Poisson-QCH

Conclusion

1. Increased performance for iterative models.
2. Dependant only on the payoff of the game.

   → Generally applicable to any domain
3. The belief that Level-0 agents only exist in the minds of higher level agents should be questioned.
4. Non-strategic behaviour is an important aspect of human behaviour.

Proposed Architecture



Sources

[1] James R. Wright and Kevin Leyton-Brown (2019): "Level-0 Models for Predicting Human Behavior in Games"

[2] J. Hartford, J. Wright, K. Leyton-Brown (2016) - “Deep Learning for Predicting Human Strategic Behavior.“

[3] K. Leyton-Brown,J. Wright (2014), Level-0 Meta-Models for Predicting Human Behavior in Games (Slides) (cs.ubc.ca/~kevinlb/talk.php?u=2014-Level0.pdf)

[4] nytimes.com/interactive/2015/08/13/upshot/are-you-smarter-than-other-new-york-times-readers.html

[5] kristiannanagorcka.com/wp-content/uploads/2014/07/Depositphotos_12705383_s.jpg

[6] utmb.edu/images/librariesprovider84/default-album/strategic-planning.jpg?sfvrsn=ea098358_2

[7]driving-tests.org/wp-content/uploads/2012/02/back-parking.jpg