Introduction to Reinforcement Learning

Artificial Intelligence for Games

Denis Zavadski

Overview

- Introduction
- Model Based Learning
- Model free Learning
  - Monte Carlo
  - Temporal Difference
- Function Approximation

Motivation

- Training without a supervisor, only with reward
- Feedback not always instantly received
- Acting in environments, where actions have an impact on subsequent data

  → Non i.i.d. data

Definitions

- Reward R_t , (G_t)
- Action $a_t \in A$
- State $s_t \in S$
- Model
- MDP
- Policy π
- Value Function v
- Action Value Function q

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- scalar feedback
- return G_t

$$G_t = R_t + \gamma R_{t+1} \cdots = \sum_{k=1}^{\infty} \gamma^k R_{t+k}$$

- discount $\gamma \in [0, 1]$

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- environment state
- agent state
- Markov State

$$\mathbb{P}[S_{t+1} \mid S_t] = \mathbb{P}[S_{t+1} \mid S_1, \ldots, S_t]$$

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- predicts environment by
  - predicting next state

$$\mathcal{P}^a_{s's} = \mathbb{P}[s_{t+1} = s' \mid s_t = s, a_t = a]$$

  - predicting next Reward

$$\mathcal{R}^a_s = \mathbb{E}[R'_t \mid s_t = s, a_t = a]$$

- optional

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- Markov Decision Process is a formal environment representation
- tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$
- discount $\gamma \in [0, 1]$

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- defines agents behaviour
- maps from states to actions
  - Deterministically $\pi(s) = a$
  - Stochastically $\pi(a|s) = \mathbb{P}[a|s]$

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- the state-value-function is the expected return starting from state s and following policy π

$$v_\pi(s) = \mathbb{E}_\pi[G_t \mid s]$$

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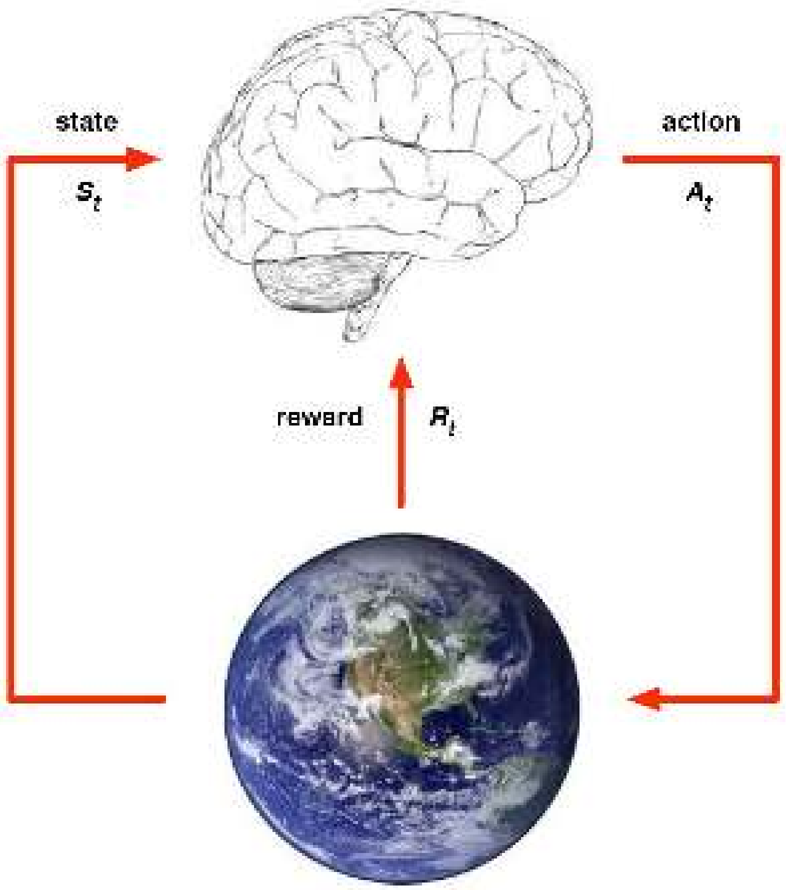
$$q_\pi(s, a) = \mathbb{E}_\pi[G_t \mid s, a]$$

Interaction with Environment



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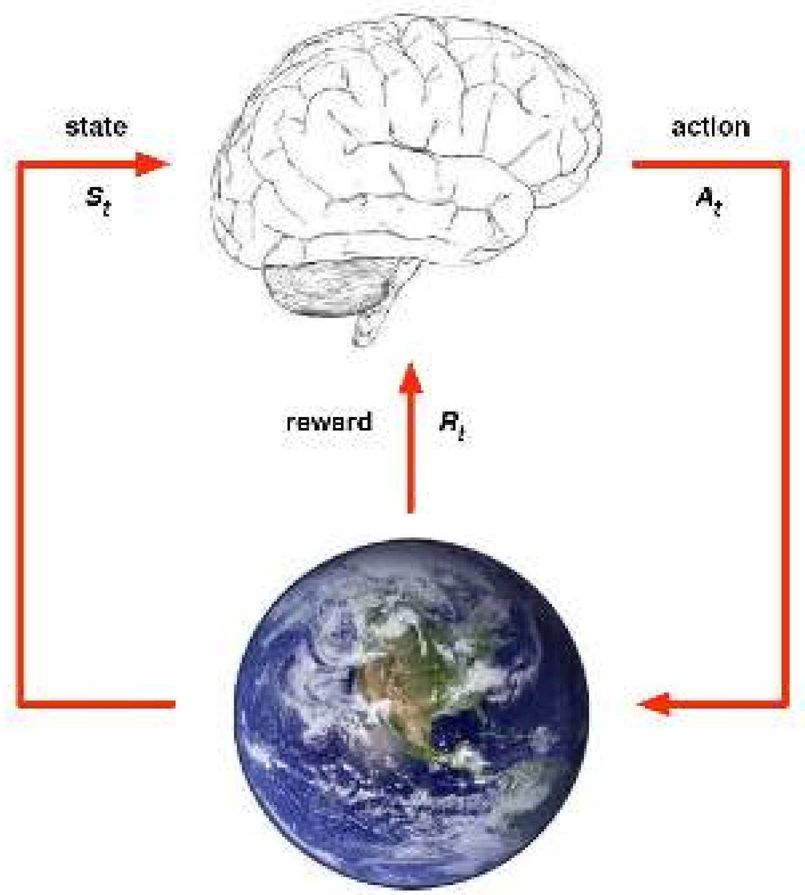

Tasks

- Prediction
  - Evaluating given policy
- Control
  - Finding best policy

Characteristics

- Value based
- Policy based

- Model based
- Model free

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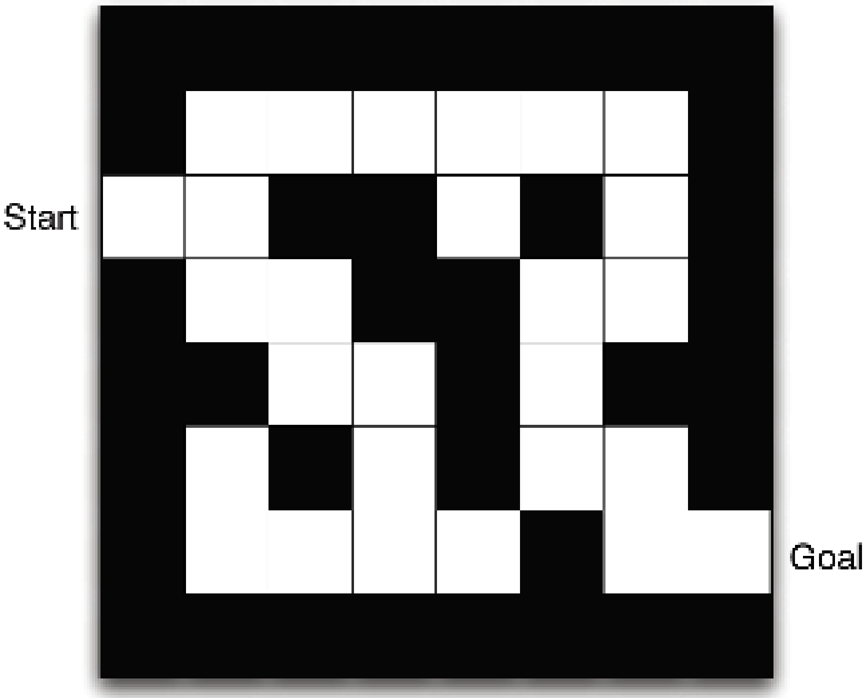
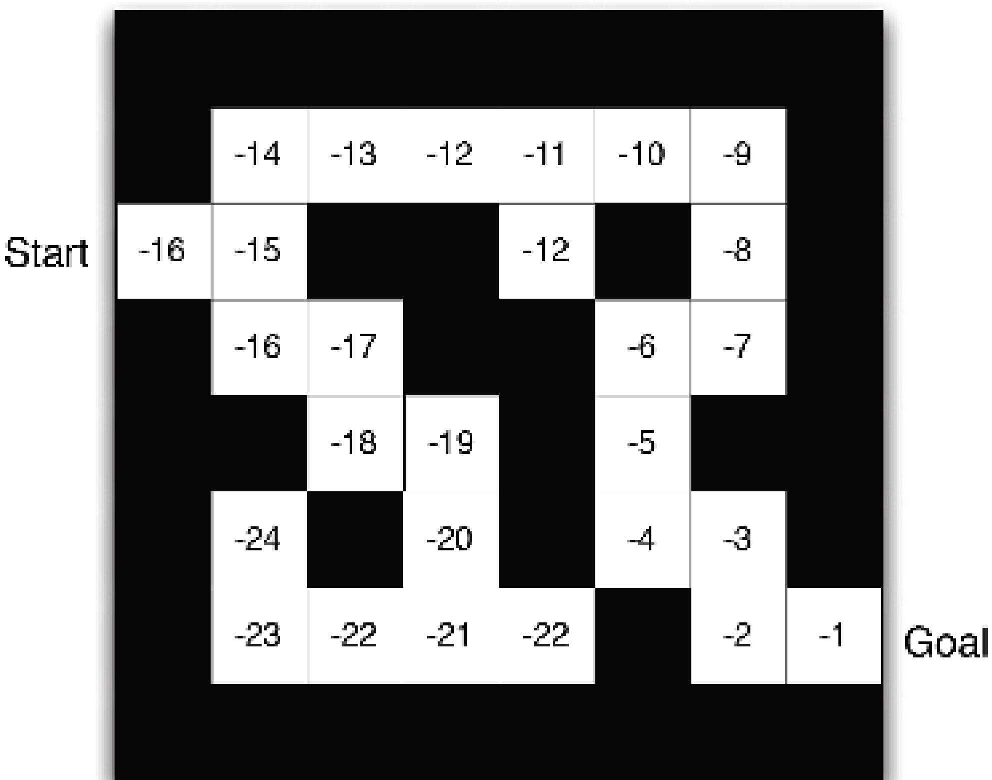


- Rewards: -1/step
- States: location

Value Based vs. Policy Based



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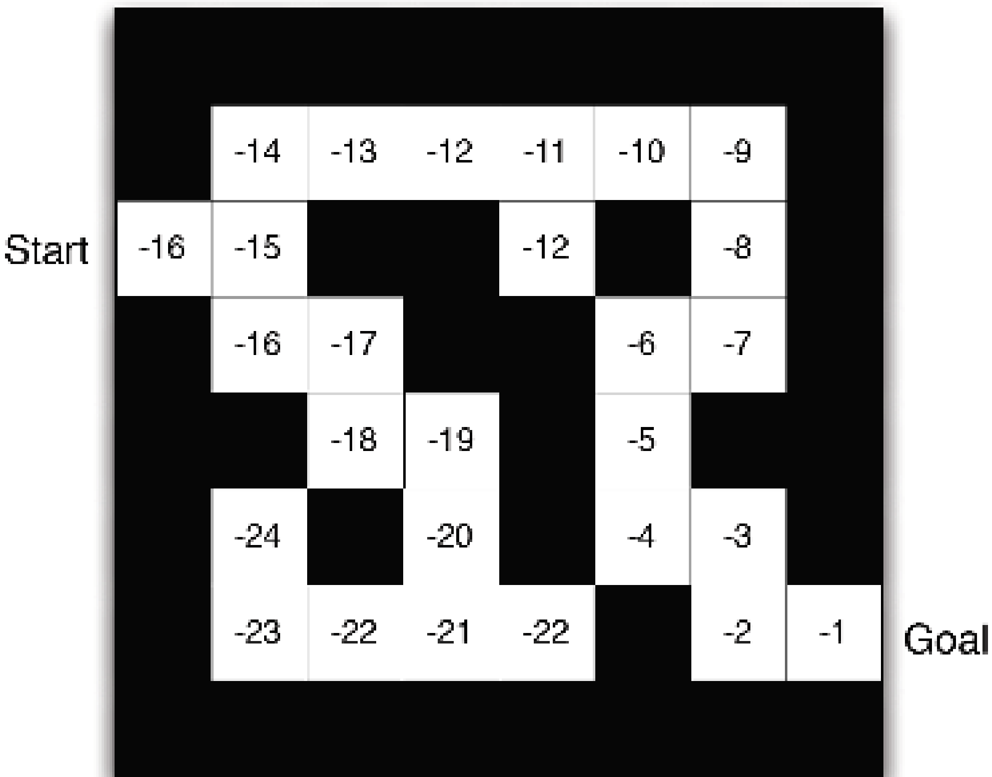

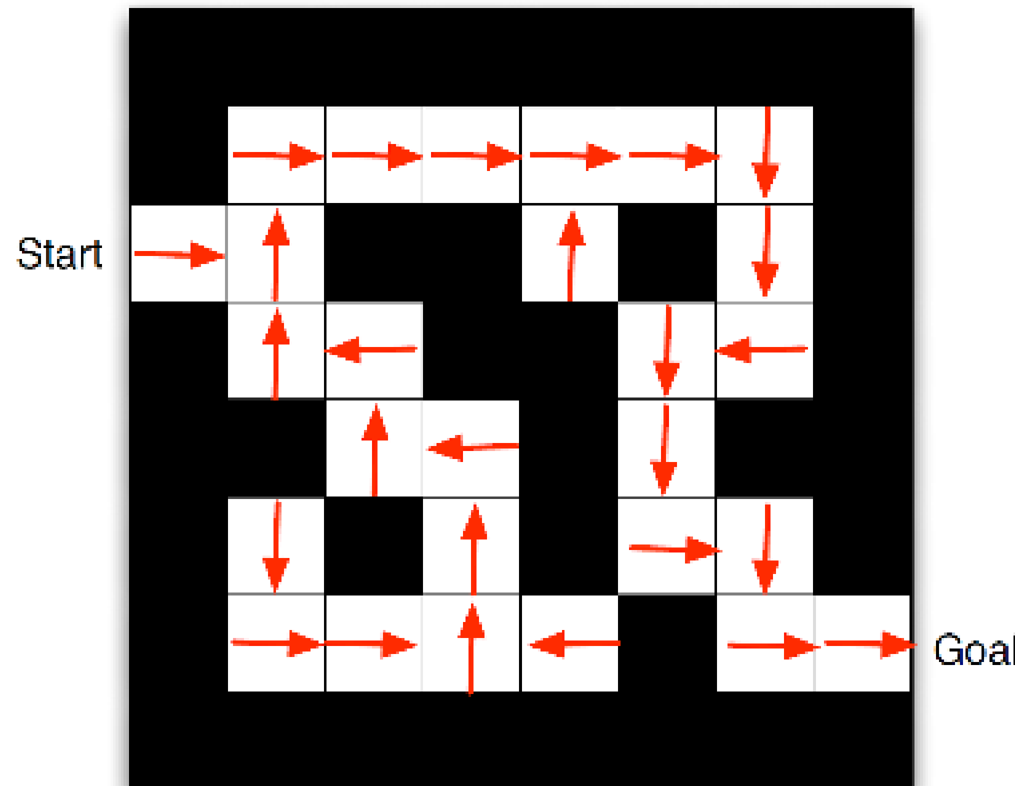

Bellman Equation

$$v(s)_\pi = \mathbb{E}_\pi[G_t \mid s]$$

Bellman Equation

$$
\begin{aligned}
v(s)_\pi =& \mathbb{E}_\pi[G_t \mid s] \\
=& \mathbb{E}_\pi[R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \dots \mid s] \\
=& \mathbb{E}_\pi[R_t + \gamma (R_{t+1} + \gamma R_{t+2} + \dots) \mid s] \\
=& \mathbb{E}_\pi[R_t + \gamma v_\pi(s_{t+1}) \mid s]
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- matrix notation

$$v = \mathcal{R} + \gamma \mathcal{P} v$$

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$$\begin{aligned} v &= \mathcal{R} + \gamma \mathcal{P} v \\ (\mathbb{I} - \gamma \mathcal{P}) v &= \mathcal{R} \\ v &= (\mathbb{I} - \gamma \mathcal{P})^{-1} \mathcal{R} \end{aligned}$$

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- analogous for *q*

Optimal Functions

- Optimal value function

$$v_*(s) = \max_\pi v_\pi(s)$$

- Optimal value action function

$$q_*(s,a) = \max_\pi q_\pi(s,a)$$

- Optimal policy

$$\pi_* \geq \pi \quad, \forall \pi$$

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Theorem

- There exists an optimal policy
- The opt. Policy always archieves the optimal state-(action-)functions

Policy Iteration

- evaluate value functions iteratively

$$v^{k+1} = \mathcal{R}^{\pi} + \gamma \mathcal{P}^{\pi} v^{k}$$

- Improve the policy by acting greedily w.r.t. v

$$\pi^{new} = greedy(v_{\pi})$$

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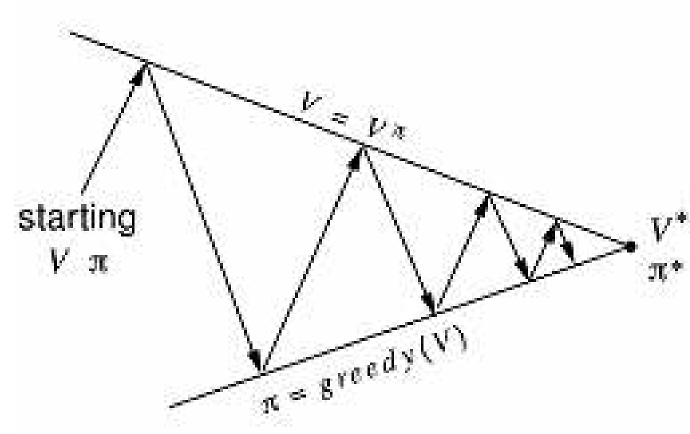

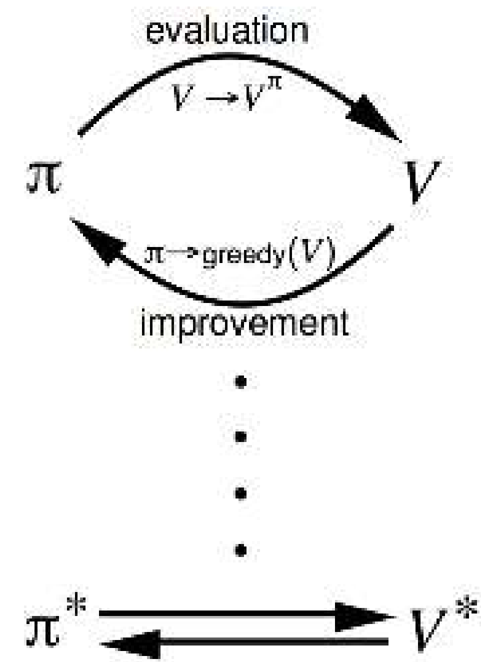

Value Iteration

- like one-step policy iteration without policy

$$v^{k+1} = \max_a \mathcal{R}^a + \gamma \mathcal{P}^a v^k$$

- pseudo policy is to act greedily

Monte Carlo Policy Evaluation

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$$
\begin{aligned}
\mu_k &= \frac{1}{k}\sum_{j=1}^{k} x_j = \frac{1}{k}\left(x_k + \sum_{j=1}^{k-1} x_j\right) \\
&= \frac{1}{k}(x_k + (k-1)\mu_{k-1}) \\
&= \mu_{k-1} + \frac{1}{k}(x_k - \mu_{k-1})
\end{aligned}
$$

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$$= \frac{1}{k}(x_k + (k-1)\mu_{k-1})$$
$$= \mu_{k-1} + \frac{1}{k}(x_k - \mu_{k-1})$$

- Update after each episode:

$$N(s_t) \longleftarrow N(s_t) + 1$$
$$v(s_t) \longleftarrow v(s_t) + \frac{1}{N(s_t)}(G_t - v(s_t))$$
$$\longleftarrow v(s_t) + \alpha(G_t - v(s_t))$$

for each state s_t with return G_t

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- Update values $v(s_t)$ towards estimated return $R_t + \gamma v(s_{t+1})$

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Temporal Difference TD(0)

- Is model free
- Learns from incomplete episodes
- Update values *v(s$_t$)* towards estimated return $R_t + \gamma v(s_{t+1})$

$$v(s_t) \longleftarrow v(s_t) + \alpha(R_t + \gamma v(s_{t+1}) - v(s_t))$$

- With TD target $R_t + \gamma v(s_{t+1})$
- And TD error $\delta_t = R_t + \gamma v(s_{t+1}) - v(s_t)$

MC vs. TD

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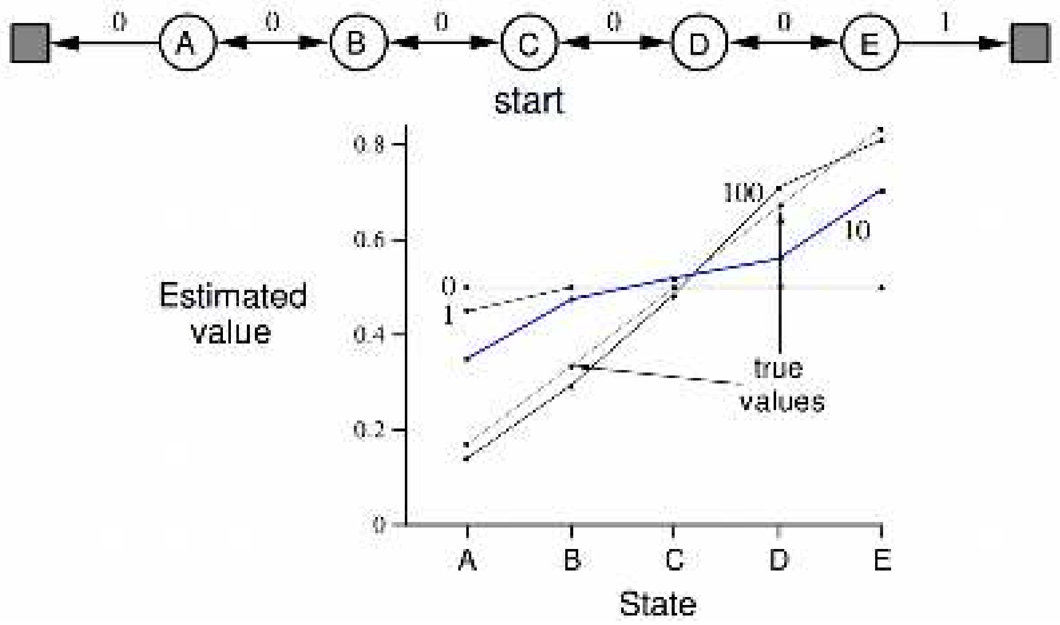
- Both converges to v^*, but differently
- TD is usually more efficient than MC

Random Walk example



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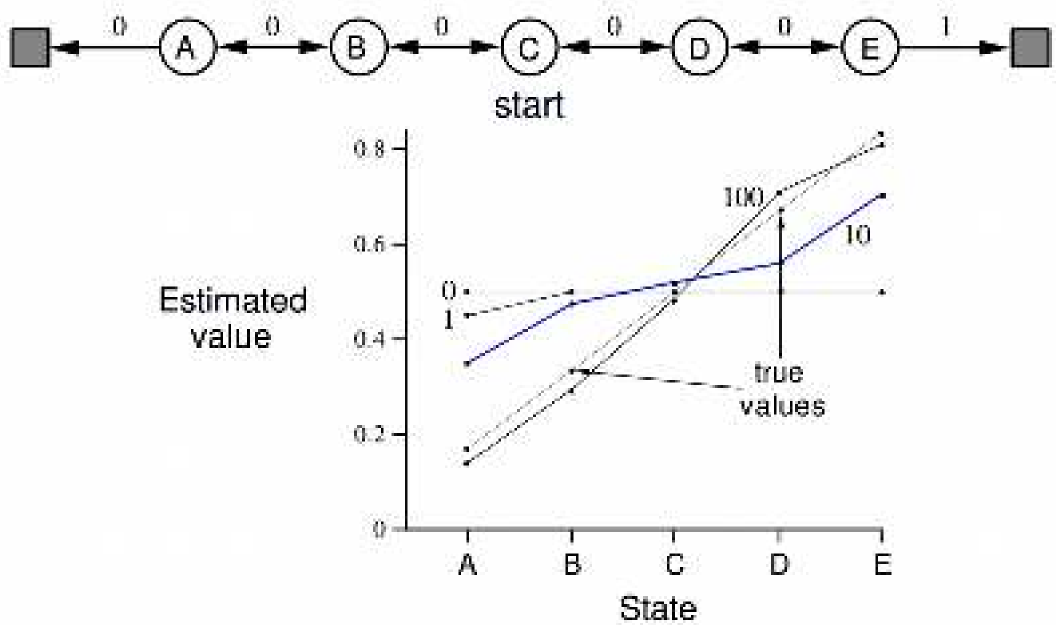

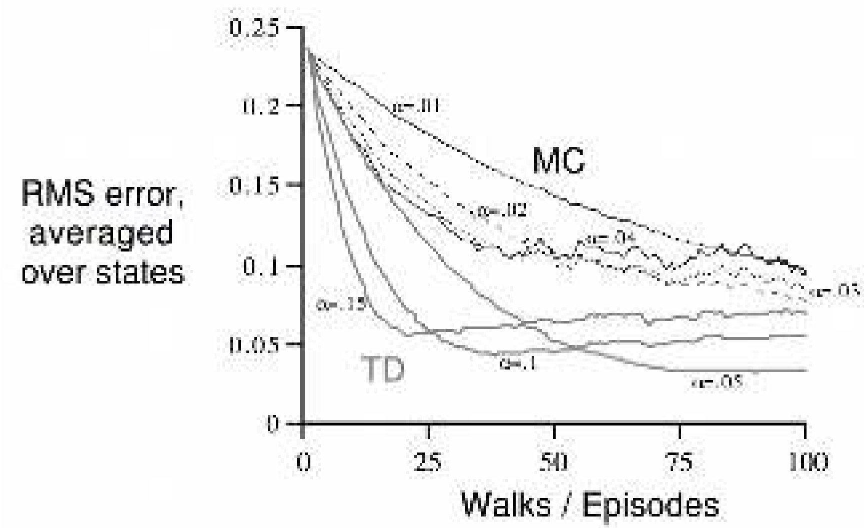

AB Example

- Two states A,B; no discounting
  8 episodes of experience

- A , 0, B, 0
  B, 1
  B, 1
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  B, 1
  B, 0

- What is v(A), v(B) ?

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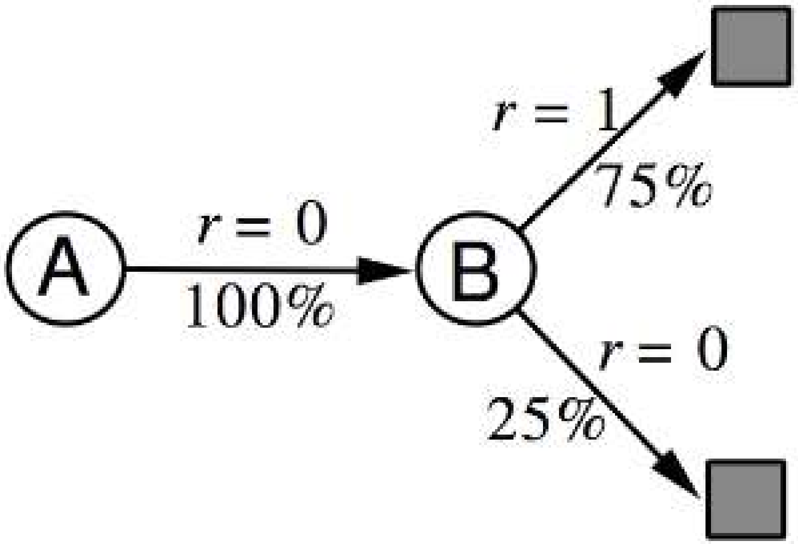
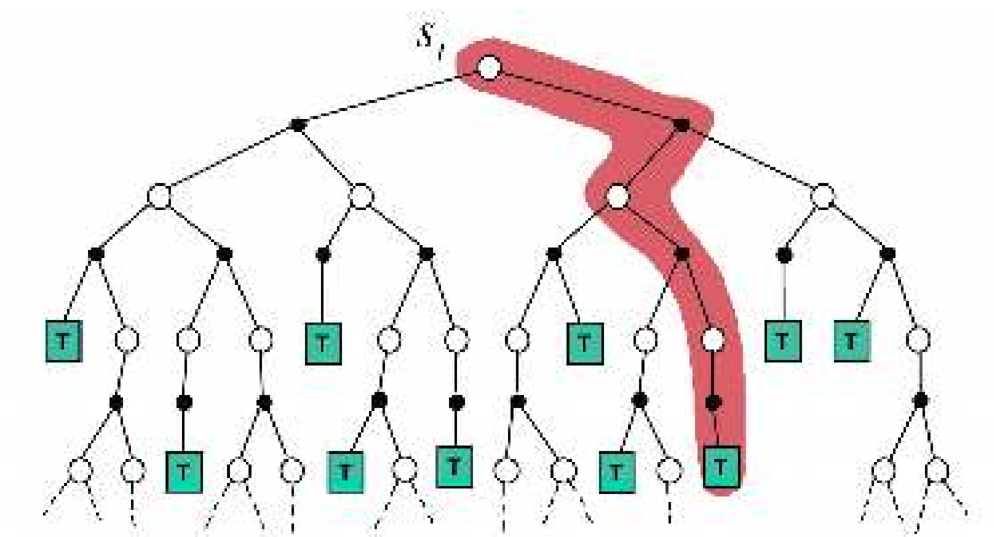


- What is v(A), v(B) ?

Updates MC, TD(0)

$$\mathrm{v}(\mathrm{s}_t) \longleftarrow v(s_t) + \alpha(G_t - v(s_t))$$



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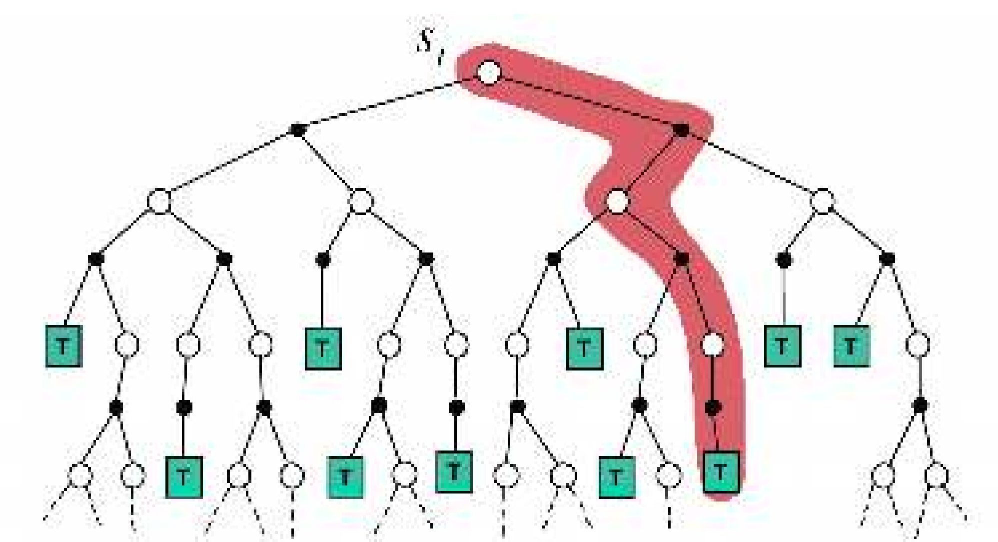

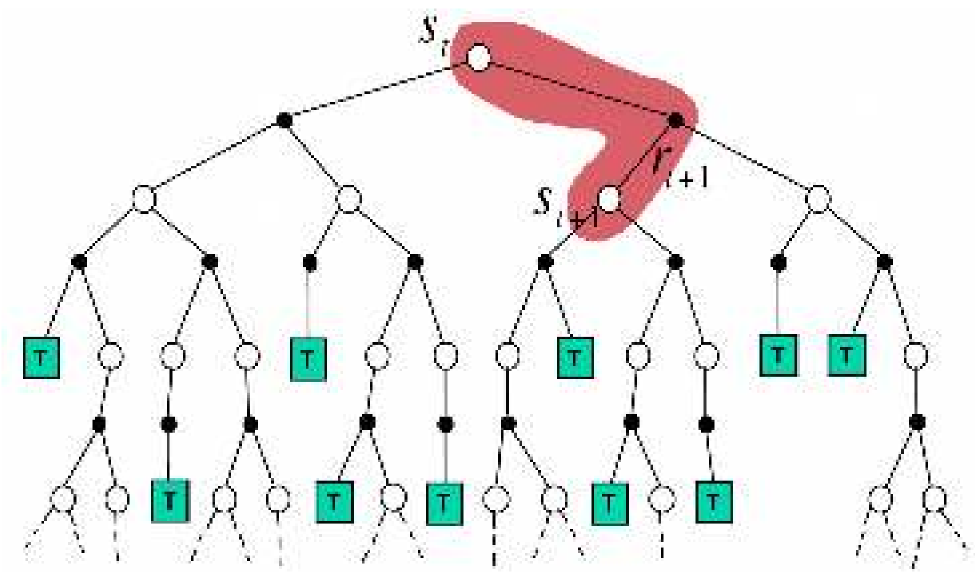

n-Step Temporal Difference

- No restriction to one-step look ahead

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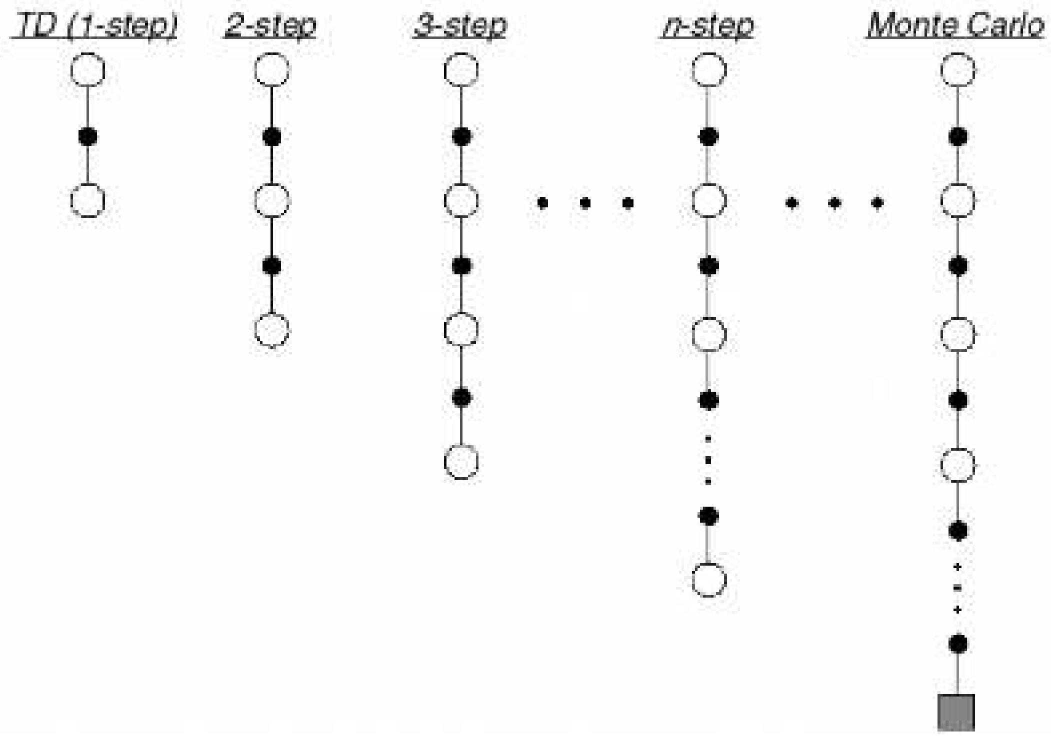
$$G_t^{(n)} = \left(\sum_{i=0}^{n-1} \gamma^i R_{t+i} \right) + \gamma^n R_n$$



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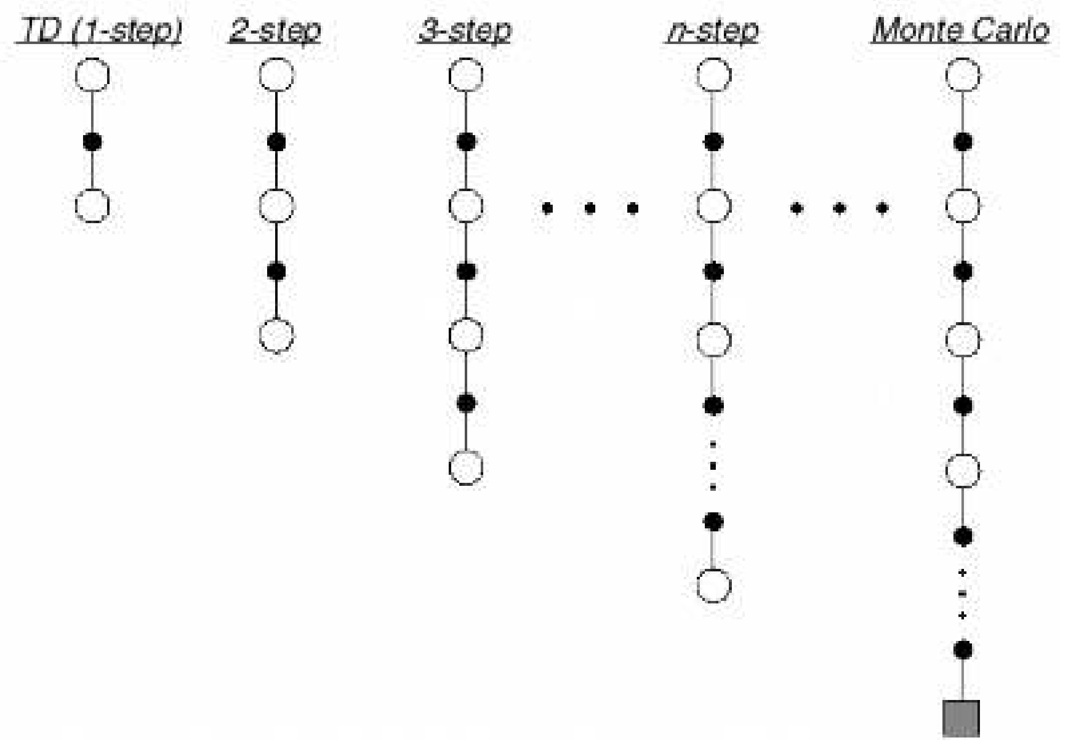

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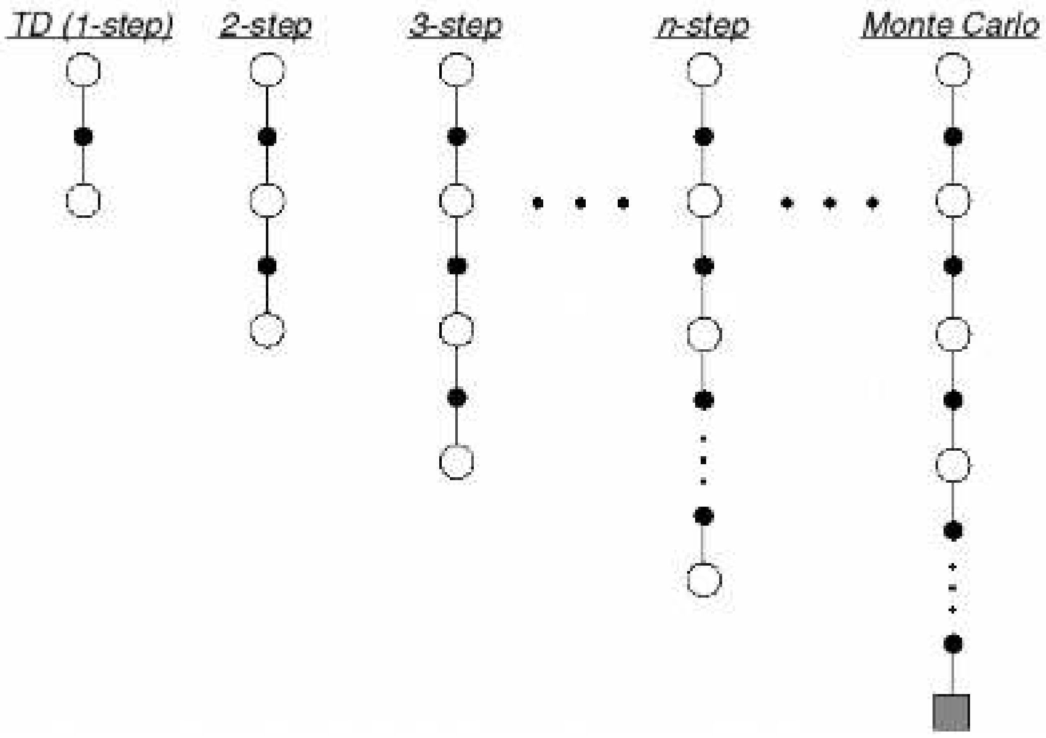
- Arbitrary averaging possible
- TD(λ) takes geometric average over all steps

$$G_t^{\lambda} = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}$$



Model-free Policy Iteration

- Problem:

  policy iteration over *v(s)* requires a model

$$\pi^{'}(s) = \arg\max_a \mathcal{R}_s^a + \gamma \mathcal{P}_{ss'}^a v(s')$$

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- Solution:

  use the action-value function *q(s, a)* instead

$$\pi^{'}(s) = \arg\max_a q(s, a)$$

Exploitation vs Exploration

Exploitation

exploits what is already known to maximize reward

Exploration

finds more information about the environment

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- agent needs to find the optimal policy

→ has to explore his possibilities

- agent has to maximize his return

→ should not give up on to much known rewards

ε-Greedy Exploration

- Chose a random action with probability *ε*
- with *m* actions available

$$\pi(a|s) = \begin{cases} \epsilon/m + 1 - \epsilon & \text{if } a^* = \arg\max_a q(s,a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

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- same as before, but with action-value function q

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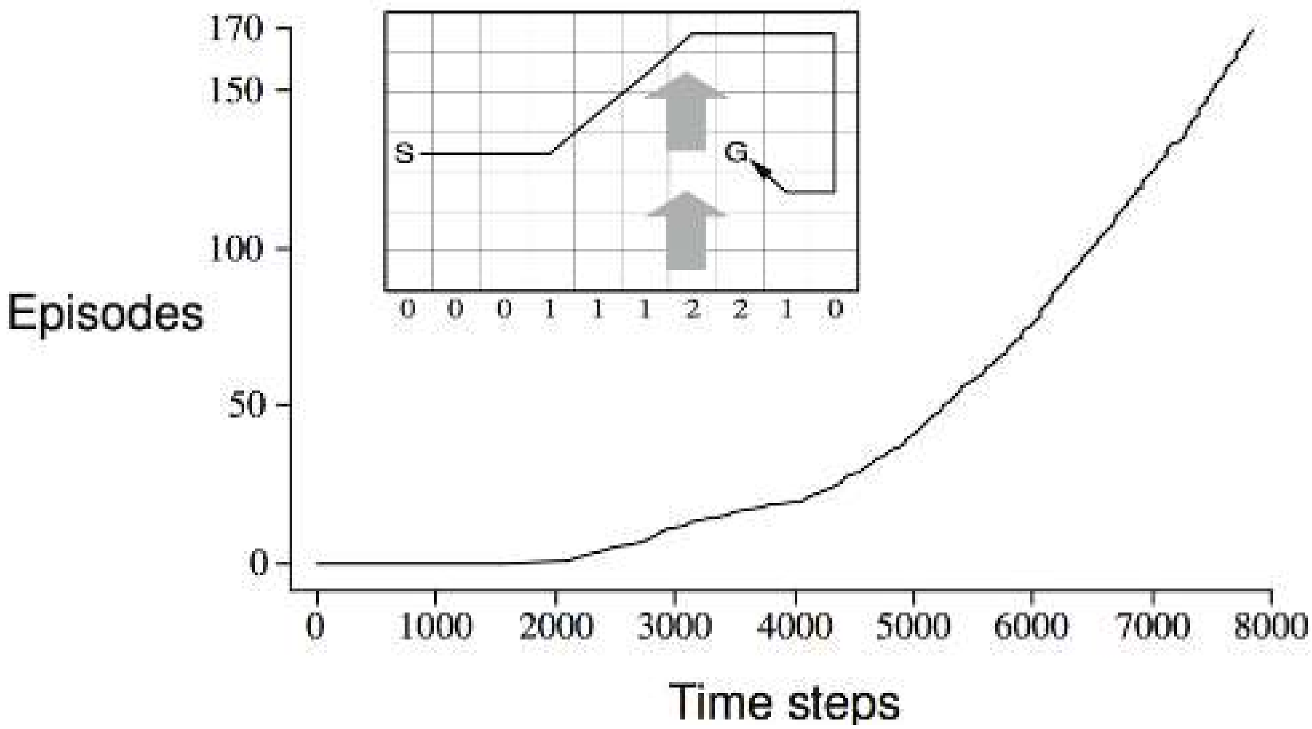
Typical Improvement (SARSA)

- seemingly no improvement at beginning



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- **Solution:**

  → approximate the value functions instead of storing them

Value Function Approximation

- Represent state by feature vector

$$x(s) = (x_1(s), \ldots, x(_n(s))^T$$

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$$\mathcal{L}(w) = \mathbb{E}_\pi[(v_\pi(s) - \hat{v}_\pi(s, w))^2]$$

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- Gradient of MSE w.r.t. *w*

$$\nabla_w \mathcal{L}(w) = -2\mathbb{E}_\pi[v_\pi(s) - \hat{v}_\pi(s, w)]\nabla_w \hat{v}(s, w)$$

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$$\mathcal{L}(w) = \mathbb{E}_\pi[(v_\pi(s) - \hat{v}_\pi(s, w))^2]$$

- Gradient of MSE w.r.t. *w*

$$\nabla_w \mathcal{L}(w) = -2\mathbb{E}_\pi[v_\pi(s) - \hat{v}_\pi(s, w)]\nabla_w \hat{v}(s, w)$$

- Gradient update *Δw*

$$\Delta w = -\frac{1}{2}\alpha \nabla_w \mathcal{L}(w)$$

Value Function Approximation

- Represent state by feature vector

$$x(s) = (x_1(s), \ldots, x(_n(s))^T$$

- Represent value function by linear combination of features

$$\hat{v}(s, w) = x(s)^T w$$

- Minimize mean squared error between true value and prediction by gradient descent

$$\Delta w = \alpha \mathbb{E}_\pi [v_\pi(s) - \hat{v}_\pi(s, w)] x(s)$$

- MSE

$$\mathcal{L}(w) = \mathbb{E}_\pi [(v_\pi(s) - \hat{v}_\pi(s, w))^2]$$

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$$\Delta w = \alpha \mathbb{E}_\pi [v_\pi(s) - \hat{v}_\pi(s, w)] x(s)$$

- Update weights with stochastic gradient descent

$$\Delta w = \alpha (v_\pi(s) - \hat{v}_\pi(s, w)) x(s)$$

- MSE

$$\mathcal{L}(w) = \mathbb{E}_\pi [(v_\pi(s) - \hat{v}_\pi(s, w))^2]$$

- Gradient of MSE w.r.t. *w*

$$\nabla_w \mathcal{L}(w) = -2\mathbb{E}_\pi [v_\pi(s) - \hat{v}_\pi(s, w)] \nabla_w \hat{v}(s, w)$$

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Updates for MC, TD(0), TD(λ)

$$\Delta w = \alpha(v_\pi(s) - \hat{v}_\pi(s, w))x(s)$$

Substitute the target with the corresponding returns

Updates for MC, TD(0), TD(λ)

$$\Delta w = \alpha(v_\pi(s) - \hat{v}_\pi(s, w))x(s)$$

Substitute the target with the corresponding returns

- MC $\quad v(s) \longrightarrow G_t$

$$\Delta w = \alpha(G_t - \hat{v}_\pi(s, w))x(s)$$

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- TD(0) $\quad v(s_t) \longrightarrow R_t + \gamma \hat{v}(s_{t+1}, w)$

$$\Delta w = \alpha(R_t + \gamma \hat{v}(s_{t+1}, w) - \hat{v}_\pi(s_t, w))x(s_t)$$

Updates for MC, TD(0), TD(λ)

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- TD(λ) $\quad v(s) \longrightarrow G_t^\lambda$

$$\Delta w = \alpha(G_t^\lambda - \hat{v}_\pi(s, w))x(s)$$

Action-Value Function Approx.

- Represent state by feature vector

$$x(s,a) = (x_1(s,a), \ldots, x(_n(s,a))^T$$

Action-Value Function Approx.

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$$\Delta w = \alpha(q_\pi(s,a) - \hat{q}_\pi(s,a,w))x(s,a)$$

Updates for MC, TD(0), TD(λ)

$$\Delta w = \alpha(q_\pi(s,a) - \hat{q}_\pi(s,a,w))x(s,a)$$

Substitute the target with the corresponding returns

Updates for MC, TD(0), TD(λ)

$$\Delta w = \alpha(q_\pi(s, a) - \hat{q}_\pi(s, a, w))x(s, a)$$

Substitute the target with the corresponding returns

- MC $\qquad q(s, a) \longrightarrow G_t$

$$\Delta w = \alpha(G_t - \hat{q}_\pi(s, a, w))x(s, a)$$

Updates for MC, TD(0), TD(λ)

$$\Delta w = \alpha(q_\pi(s,a) - \hat{q}_\pi(s,a,w))x(s,a)$$

Substitute the target with the corresponding returns

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Substitute the target with the corresponding returns

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$$\Delta w = \alpha(R_t + \gamma \hat{q}(s_{t+1},a_{t+1},w) - \hat{q}_\pi(s_t,a_t,w))x(s_t,a_t)$$

- TD(λ) $\quad q(s,a) \longrightarrow G_t^\lambda$

$$\Delta w = \alpha(G_t^\lambda - \hat{q}_\pi(s,a,w))x(s,a)$$

Thank you for your attention

Sources

- David Silver, UCL Course on RL

  http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching.html

  Lectures 1 to 6