Fallacies of Reasoning

How We Fool Ourselves

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7^{th} November, 2019

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Base-Rate / Prosecutor's Fallacy

Introduction

- "Humans are 54% accurate at recognizing lies."

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- What is the corresponding base-rate of lies?

Base-Rate / Prosecutor’s Fallacy

HIV Self-Test

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Negative	0	0.998

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- "[...] test result is positive. You are probably HIV positive."

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- "Accuracy greater than 99%"
- "[...] test result is positive. You are probably HIV positive."
- "This means a positive result will be correct 998 out of every 1000 tests."

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Bayes' Theorem:

$$p(\text{HIV}|\text{P}) = \frac{p(\text{P}|\text{HIV})\, p(\text{HIV})}{p(\text{P})}$$

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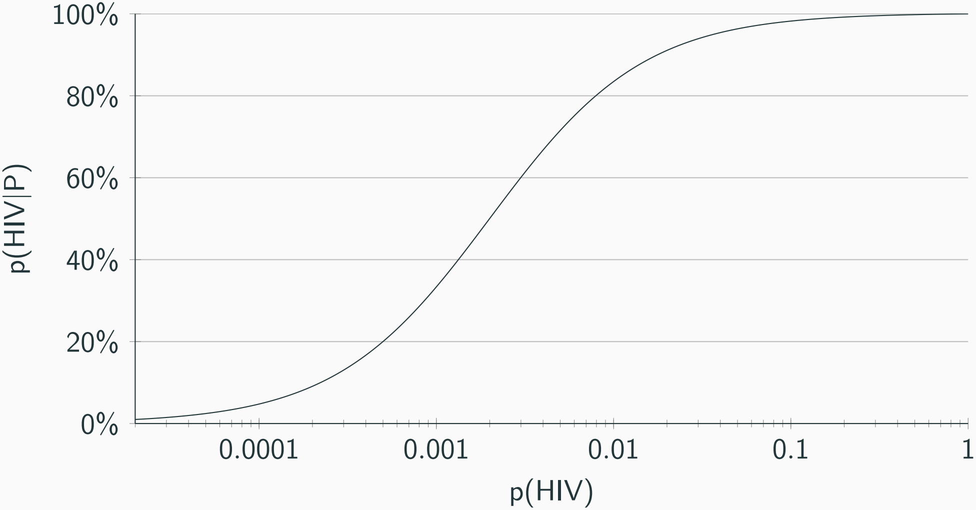
$$= \frac{p(\mathrm{P}|\mathrm{HIV})\, p(\mathrm{HIV})}{p(\mathrm{P}|\mathrm{HIV})\, p(\mathrm{HIV}) + p(\mathrm{P}|\neg\mathrm{HIV})\, p(\neg\mathrm{HIV})}$$

HIV Self-Test

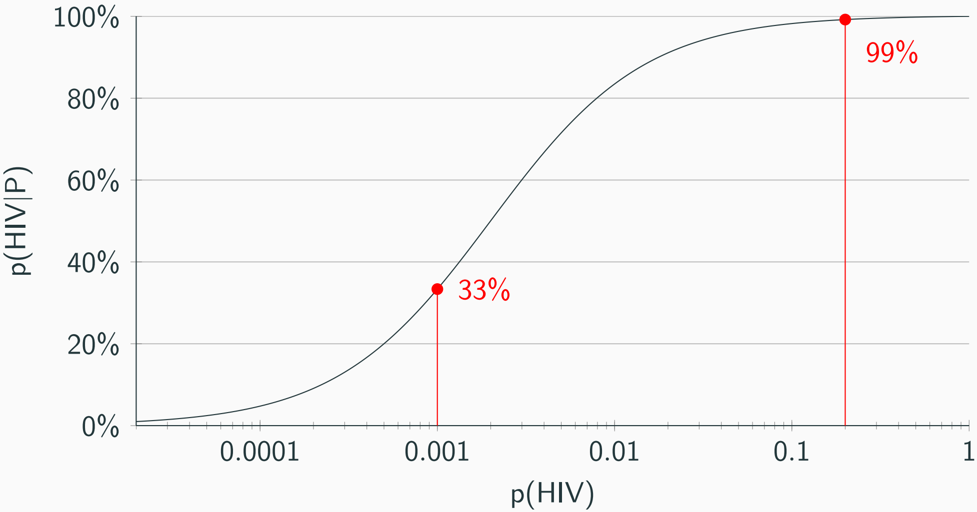
Bayes' Theorem:

$$
\begin{aligned}
\mathrm{p}(\mathrm{HIV}|\mathrm{P}) &= \frac{\mathrm{p}(\mathrm{P}|\mathrm{HIV})\,\mathrm{p}(\mathrm{HIV})}{\mathrm{p}(\mathrm{P})} \\
&= \frac{\mathrm{p}(\mathrm{P}|\mathrm{HIV})\,\mathrm{p}(\mathrm{HIV})}{\mathrm{p}(\mathrm{P}|\mathrm{HIV})\,\mathrm{p}(\mathrm{HIV}) + \mathrm{p}(\mathrm{P}|\neg\mathrm{HIV})\,\mathrm{p}(\neg\mathrm{HIV})} \\
&= \left(1 + \frac{\mathrm{p}(\mathrm{P}|\neg\mathrm{HIV})}{\mathrm{p}(\mathrm{HIV})}\,(1 - \mathrm{p}(\mathrm{HIV}))\right)^{-1}
\end{aligned}
$$

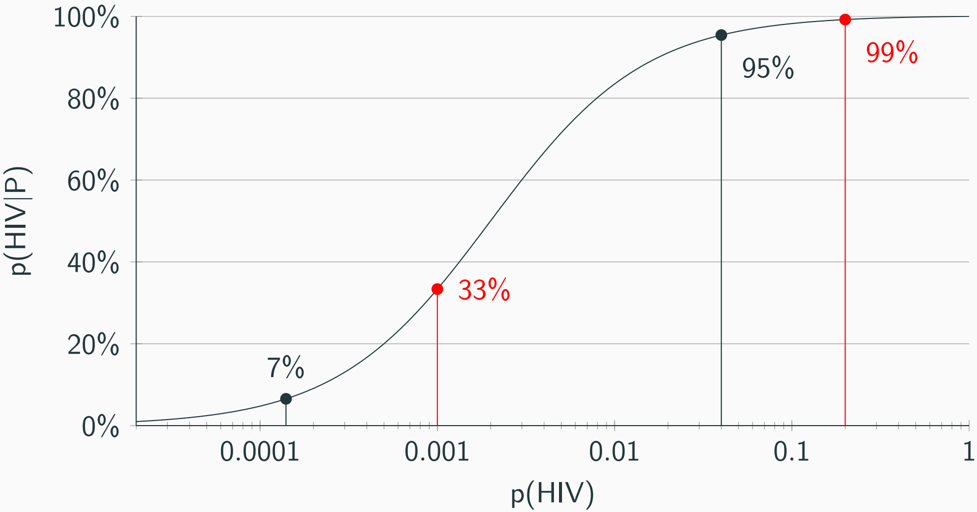
HIV Self-Test



HIV Self-Test



HIV Self-Test



Base-Rate / Prosecutor’s Fallacy

SIDS

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- Sally Clark; both children, in 1996 and 1998, died of SIDS
- Prosecutor: "The chance for that is only one in 73 Million, so she killed them!"
- She was tried and convicted, spent three years in prison before her eventual acquittal

SIDS

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- Probability of her having killed them about 10%

Base-Rate / Prosecutor's Fallacy

p-Values

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- $\text{p-Value} = p(\text{Data}|\text{Null})$

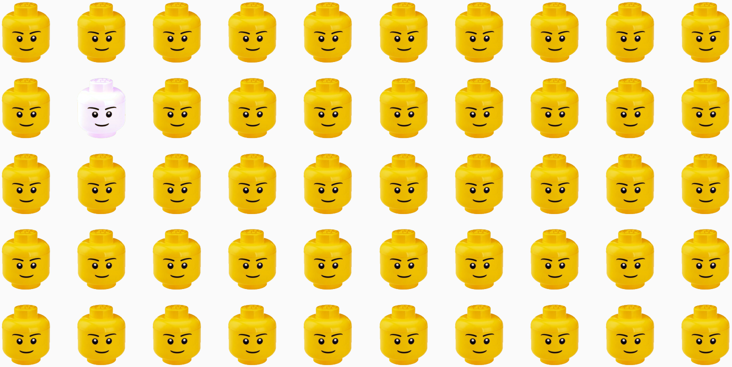
p-Values

- p-Value $= \text{p(Data|Null)} \neq \text{p(Null|Data)}$

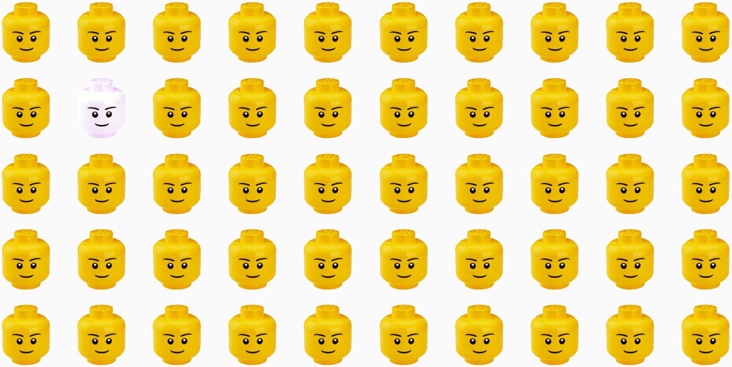
p-Values

- p-Value $=$ p(Data$|$Null) $\neq$ p(Null$|$Data)
- "Extraordinary claims require extraordinary evidence"

p-Values



p-Values



$$p \approx 1/400 = 0.0025 < 0.05$$

Gambler's / Hot-Hand Fallacy

Monte Carlo 1913

Gambler's / Hot-Hand Fallacy

Useless Advice

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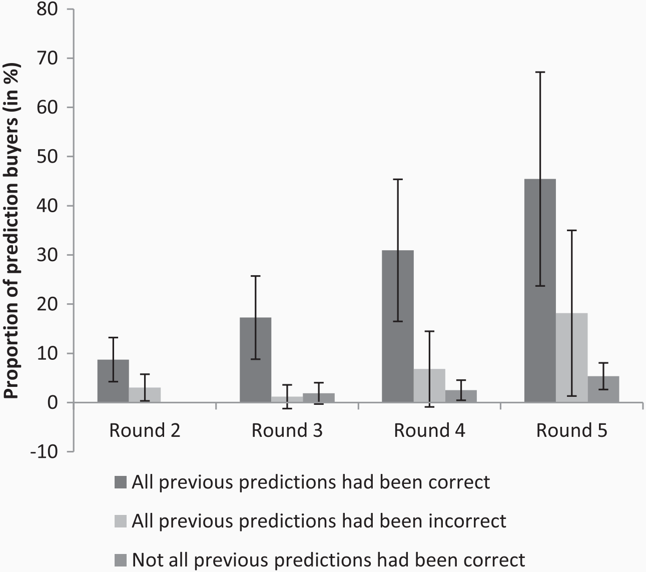
- It was completely obvious that the outcome was only determined by chance
- They were given a prediction for every round, and could pay to open it before the coin toss
- They were always told to open it after every toss

Useless Advice



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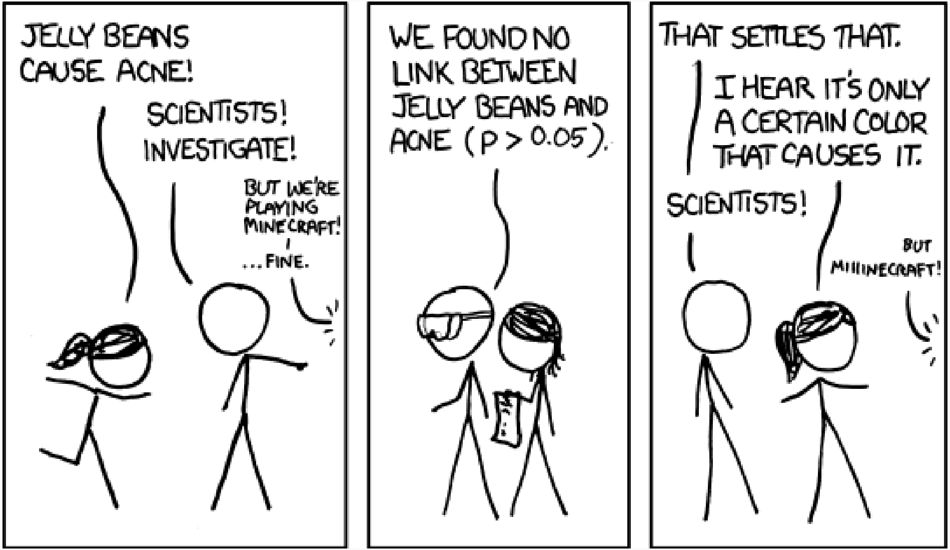

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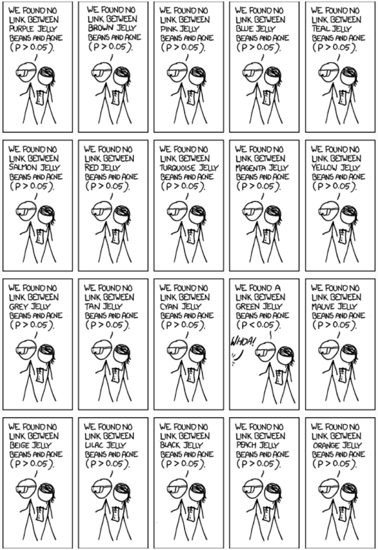

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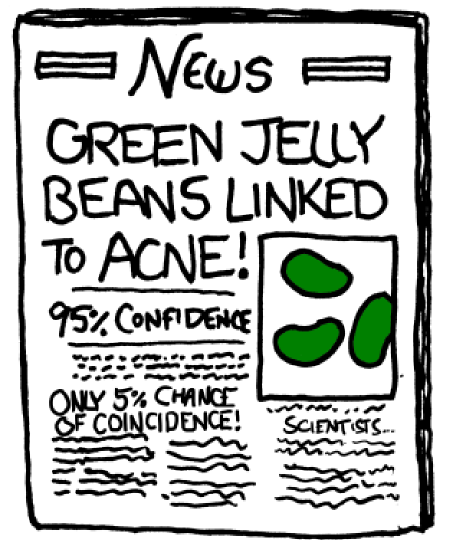

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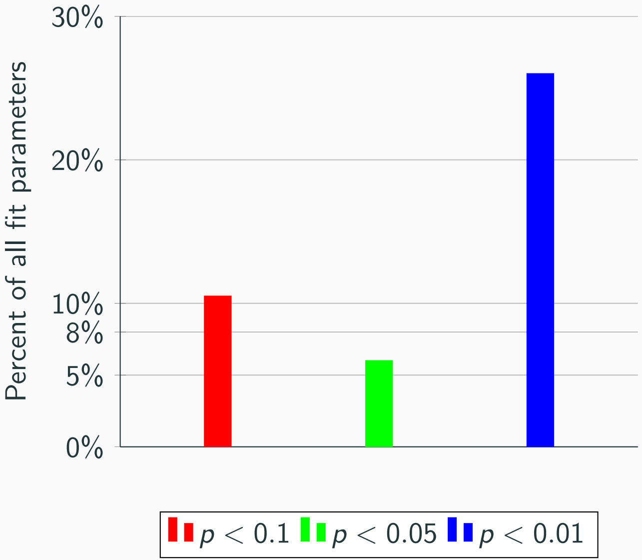
- Here, this would mean $p < 0.0003$
- Very conservative

Useless Advice



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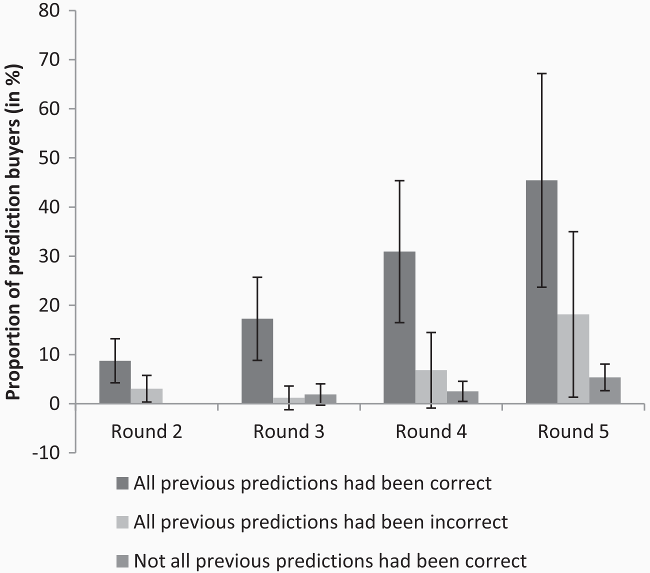

Hindsight Bias

Captain Hindsight

Hindsight Bias

Clinicopathologic Conferences

CPC

- A presenter goes through an old case, presents the medical information and possible diagnoses

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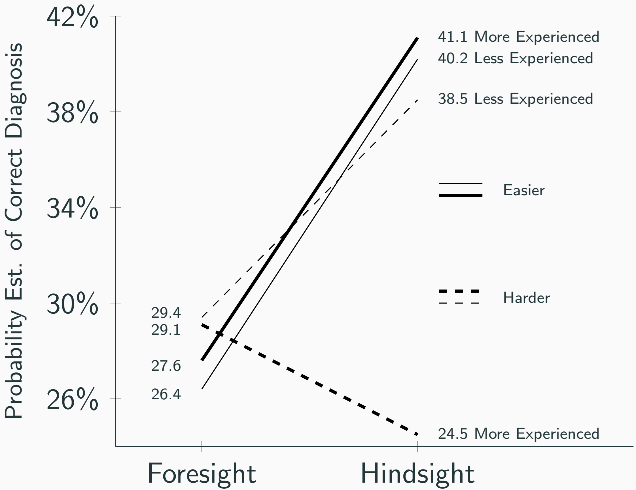
- A presenter goes through an old case, presents the medical information and possible diagnoses
- Afterwards, the pathologist announces the real diagnosis
- 4 cases, 2 easier, 2 harder
- 160 (total) physicians were asked to estimate the prior probabilities for the diagnoses either in foresight or in hindsight

CPC



Probability Est. of Correct Diagnosis
42%
38%
34%
30%
26%
29.4
29.1
27.6
26.4
41.1 More Experienced
40.2 Less Experienced
38.5 Less Experienced
Easier
Harder
24.5 More Experienced
Foresight
Hindsight

CPC

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- Three groups: 30% $\rightarrow$ 50%

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- Three groups: 30% $\rightarrow$ 50%
- All: 35% $\rightarrow$ 45%

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- "[...] among the three groups that demonstrated the bias, 75% of hindsight physicians have higher estimates [...]. A 50% rate would be expected by chance alone."
- Three groups: 30% $\rightarrow$ 50%
- All: 35% $\rightarrow$ 45%
- Used the value for three groups in their abstract, but without the caveat!

CPC

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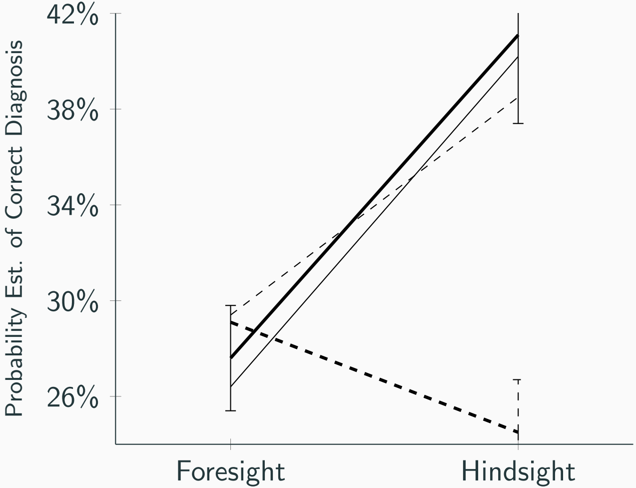
- Again, $p < 0.05$ but multiple comparisons were done
- "[...] among the three groups that demonstrated the bias, 75% of hindsight physicians have higher estimates [...]. A 50% rate would be expected by chance alone."
- Three groups: 30% $\rightarrow$ 50%
- All: 35% $\rightarrow$ 45%
- Used the value for three groups in their abstract, but without the caveat!
- $N \approx 20$ for every value in plot

CPC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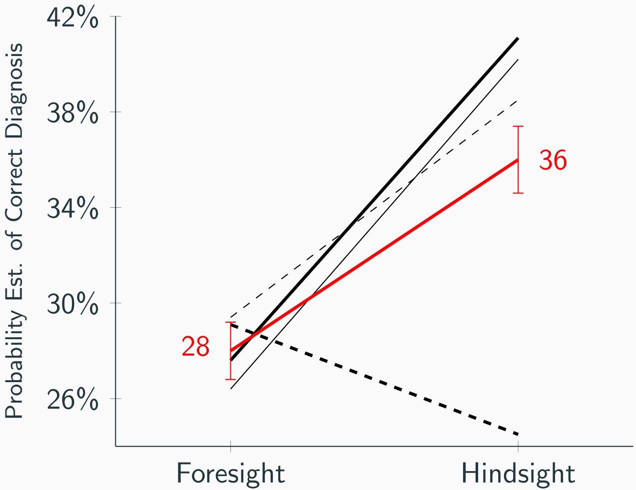
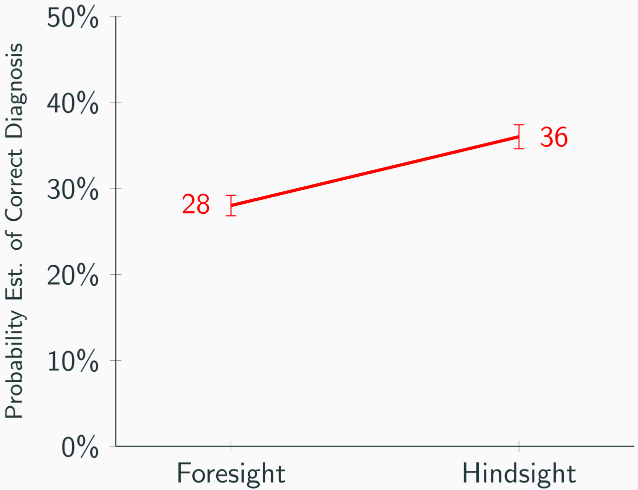
Probability Est. of Correct Diagnosis
Foresight
Hindsight

CPC



Probability Est. of Correct Diagnosis
42%
38%
34%
30%
26%
28
36
Foresight
Hindsight

CPC



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0%
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Foresight
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Hindsight Bias

Determinations of Negligence

Negligence

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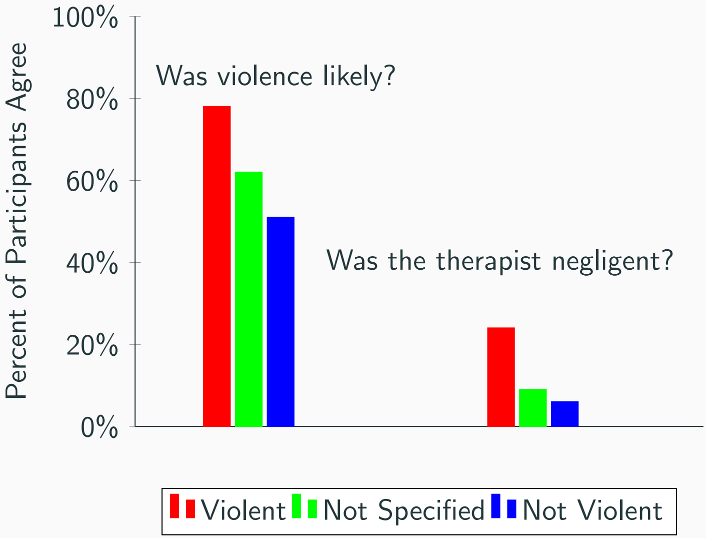
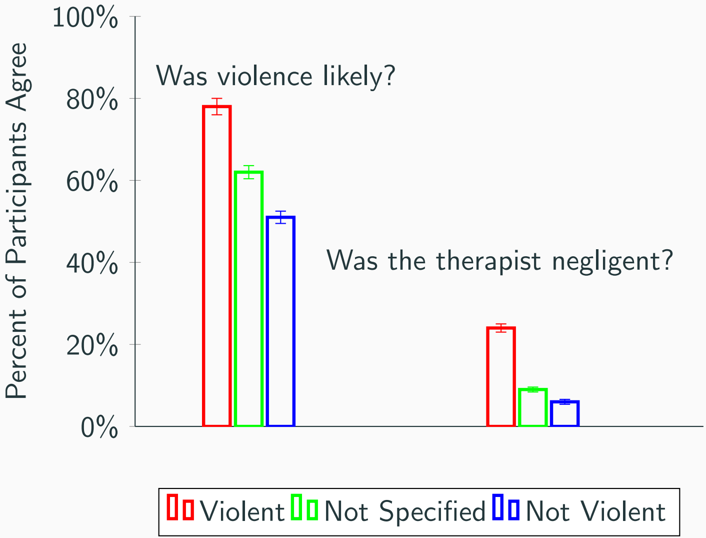
- They varied whether they reported any outcome, and if, whether violence did or did not occur
- Participants were asked to judge, as a juror, whether the therapist was negligent

Negligence


100%
80%
60%
40%
20%
0%
Percent of Participants Agree
Was violence likely?
Was the therapist negligent?
Violent
Not Specified
Not Violent

Negligence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